BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 80, Number 1, January 1974

CENTRAL MULTIPLIER THEOREMS FOR COMPACT LIE GROUPS

BY RONALD R. COIFMAN AND GUIDO WEISS[1]

Communicated by Elias Stein, June 22, 1973

The purpose of this note is to describe how central multiplier theorems for compact Lie groups can be reduced to corresponding results on a maximal torus. We shall show that every multiplier theorem for multiple Fourier series gives rise to a corresponding theorem for such groups and, also, for expansions in terms of special functions.

We use the notation and terminology of N. J. Weiss [4]. Let G denote a simply connected semisimple Lie group, $\mathfrak{g}$ its Lie algebra and $\mathfrak{h}$ a maximal abelian subalgebra; P^+ the set of positive roots in $\mathfrak{h}^*$, the dual of $\mathfrak{h}$ (with respect to some order), and $(\ ,\ )$ is the inner product on $\mathfrak{h}^*$ induced by the Killing form. With $\lambda=(\lambda_1, \cdots, \lambda_l) \in \mathbf{Z}^l$ we associate the weight $\lambda=\sum_{i=1}^{l} \lambda_i \pi_i$, where π_i are the fundamental weights adapted to the simple roots. The characters χ_λ of G are then indexed by those λ with nonnegative integer coefficients. The degree d_λ of the corresponding representation is then given by

$$d_\lambda = \prod_{\alpha\in P^+} (\lambda+\beta, \alpha) \Big/ \prod_{\alpha\in P^+} (\beta, \alpha),$$

where $\beta=\frac{1}{2}\sum_{\alpha\in P^+}\alpha$. We now define the difference operator $\mathscr{D}$ on sequences m_λ, $\lambda \in \mathbf{Z}^l$, by first putting $D_\alpha m_\lambda = m_{\lambda-\alpha}-m_\lambda$ (where the root α is identified with its coordinates with respect to the basis of π_i's) and then letting

$$\mathscr{D} m_\lambda = \Big(\prod_{\alpha\in P^+} D_\alpha\Big) m_\lambda;$$

this is a difference operator of order $(n-l)/2$ ($n=\dim G$, $l=\dim \mathfrak{h}$).

A central convolution operator M on G admits a formal expansion $M \sim \sum_{\lambda_i \geqq 0} d_\lambda m_\lambda \chi_\lambda$. The sequence $\{m_\lambda\}$ is called a multiplier for $L^p(G)$ if the operator $M * f = \sum d_\lambda m_\lambda (\chi_\lambda * f)$, defined for generalized trigonometric polynomials f (see [3]), can be extended to a bounded operator on $L^p(G)$.

AMS (MOS) subject classifications (1970). Primary 43A75, 42A18.

Key words and phrases. Multipliers, maximal torus, compact Lie groups.

[1] This research was supported, in part, by U.S. Army Grant #DA-ARO-D-31-124-72-G143.