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PERIODICALLY PERTURBED CONSERVATIVE SYSTEMS

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In this note we announce a result concerning the existence of a periodic solution for a class of periodically perturbed conservative systems. Our result, in a sense, completes a series of investigations originated by W. S. Loud [4]. Also see [1], [2], [3], and [5]. Our techniques are different from those of the authors cited above.

Consider the vector differential equation

$$x'' + \operatorname{grad} G(x) = p(t) = p(t + 2\pi), \tag{1}$$

where $p \in C(R, R^n)$, $G \in C^2(R^n, R)$. This equation can be interpreted as the newtonian equation of a mechanical system subject to conservative internal forces and periodical external forces.

THEOREM 1 (LAZER [1]). *Let A and B be real constant symmetric matrices such that if $\lambda_1 \leqq \lambda_2 \leqq \cdots \leqq \lambda_n$ and $\mu_1 \leqq \mu_2 \leqq \cdots \leqq \mu_n$ denote the eigenvalues of A and B respectively then there exist integers $N_k \geqq 0$, $k = 1, \cdots, n$, such that*

$$N_k^2 < \lambda_k \leqq \mu_k < (N_k + 1)^2.$$

If, for all $a \in R^n$, $A \leqq \partial^2 G(a)/\partial x_i\, \partial x_j \leqq B$, then (1) *has at most one 2π-periodic solution.*

Our theorem establishes the existence part of the preceding theorem. More specifically, we prove

THEOREM (1)*. *If G, A and B satisfy the hypothesis of Theorem* 1, *then* (1) *has a 2π-periodic solution.*

The key to the proof of our theorem is

LEMMA 1. *Let $\bar{Q}(t)$ be a real $n \times n$ symmetric matrix whose elements are bounded, measurable and 2π-periodic on the real line. Let A and B be real constant symmetric matrices such that $A \leqq \bar{Q}(t) \leqq B$. If $\lambda_1 \leqq \cdots \leqq \lambda_n$ and $\mu_1 \leqq \cdots \leqq \mu_n$ denote the eigenvalues of A and B respectively then there*

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