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ON THE COMPACTIFICATIONS OF ARITHMETIC QUOTIENTS OF SYMMETRIC SPACES

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The purpose of this short note is to combine the results in [3] and [4] to prove the following:

THEOREM. *Let X be a bounded symmetric domain and let Γ be an arithmetically defined group of automorphisms of X. Let V^* be the Baily-Borel-Satake compactification of $V=X/\Gamma$ and let V^{**} be the set V^* with the topology defined by Piateckiĭ-Šapiro in* [5]. *The identity mapping $i: V^* \to V^{**}$ is a homeomorphism.*

PROOF. We shall use the notation in [3] and [4]. By [1], the identity mapping $i: V^* \to V^{**}$ is continuous. Since V^* is compact and Hausdorff, i is a homeomorphism if and only if V^{**} is Hausdorff. Let $j: V \to V$ be the identity. By Theorem B of [3], V is hyperbolically imbedded in V^*. The proof of Theorem 1 in [4] shows that j extends to a continuous map $j: V^{**} \to V^*$. It is important to note that the proof does not make use of the fact that V^{**} is Hausdorff. Clearly j is a continuous inverse to i since V is dense in V^*. Q.E.D.

Armand Borel has proven the previous theorem by a more general and more natural method. He can show that the topology of Satake compactifications can be defined by means of analogues of cylindrical sets and therefore without using fundamental sets. However, since it will be a while before he publishes his result, the theorem above does clarify the question concerning the equivalence of the compactifications V^* and V^{**}.

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