

COHOMOLOGY AND WEIGHT SYSTEMS FOR NILPOTENT LIE ALGEBRAS

BY G. LEGER AND E. LUKS

Communicated by Dock Rim, July 17, 1973

1. This paper announces results concerning the cohomology groups $H^*(N, N)^T$ where N is in a certain class of finite-dimensional nilpotent Lie algebras over a field k and T is an abelian Lie algebra faithfully represented as a maximal diagonalizable algebra of derivations of N ; we shall refer to such an N as a T -algebra. The additional hypotheses to be placed on the pair N, T are inspired by the case when T is a Cartan subalgebra and $T+N=B$ is a Borel subalgebra of a complex semisimple Lie algebra. In that case Kostant has shown [2] that $H^i(N, N)^T=0$ for $i \geq 2$ and the authors applied this result in [3] to conclude that $H^*(B, B)=0$. (A similar argument shows $H^*(P, P)=0$ for P parabolic.) Here we are concerned with the relations between the vanishing of $H^i(N, N)^T$, especially for $i=2$, and the structure of the algebras N .

Let W denote the set of weights of T in N . If $\dim(T)=\dim(N/N^2)=m$ then the subset of W arising from the induced representation of T on N/N^2 has precisely m elements, say $\{\alpha_1, \dots, \alpha_m\}$. Every $\alpha \in W$ then has a unique representation $\alpha = \sum c_i \alpha_i$ with each c_i a nonnegative integer and $c_i < p$ if the characteristic of k is $p > 0$. For such an α we call the sum (in \mathbb{Z}) $\sum c_i$ the height of α and denote it by $|\alpha|$. For α in W , denote by N_α the weight space for α in N .

DEFINITION. A T -algebra is called positive if

- (i) $\dim(T)=\dim(N/N^2)$,
- (ii) N is graded by the heights of the weights, i.e., if $N(j)=\bigoplus_{|\alpha|=j} N_\alpha$ then $[N(j), N(k)] \subset N(j+k)$.

REMARK. Condition (ii) is superfluous in characteristic 0. However, in characteristic $p > 0$ it has such consequences as $N^r=0$ for $r > (p-1) \cdot \dim(T)$.

2. When T is a Cartan subalgebra of a complex semisimple Lie algebra G , $T+N$ a Borel subalgebra of G and W the weights of T in N , it is well known that N is the unique positive T -algebra with corresponding weight system W . This fact is a special case of the following theorem.

AMS (MOS) subject classifications (1970). Primary 18H25, 17B30; Secondary 05C99.

Key words and phrases. Lie algebra cohomology, weight systems, deformations of Lie algebras.

Copyright © American Mathematical Society 1974