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OPTIMAL INTEGRATION-FORMULAS FOR ANALYTIC FUNCTIONS

BY R. B. BARRAR,[1] H. L. LOEB,[2] AND H. WERNER

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We prove the existence of best approximate quadrature formulas for evaluating integrals that involve analytic functions. In [1] Eckhardt and in [2], [3], [4] Richter-Dyn have considered optimal integration formulas for certain classes of Hilbert spaces of analytic functions defined as follows.

Let S be a simply connected domain in the complex plane symmetric about the real axis. Further let $\phi(z)$ be a conformal mapping from S onto the unit disk with the property that for some real $z_0 \in S$,

$$\phi(z_0) = 0, \qquad \phi'(z_0) > 0.$$

Let $[\alpha, \beta]$ be a real interval in S and $\rho(z)$ be a function defined on S so that both $\rho(z)$ and $1/\rho(z)$ are analytic in S and $\rho(z)$ is strictly positive for real z in S.

$H^2(S, \rho)$ is the Hilbert space of functions f analytic in S and satisfying

$$\int_{\partial S} |f(z)|^2 \, |\rho(z)|^2 \, |dz| < \infty,$$

where ∂S is the boundary of S. This space has the inner-product

$$(f, g) = \int_{\partial S} f(z)\bar{g}(z) \, |\rho(z)|^2 \, |dz|,$$

and the reproducing kernel [5, p. 79]

$$K(x, y) = \frac{1}{2\pi} \frac{(\phi'(x)\overline{\phi'(y)})^{1/2}}{\rho(x)\overline{\rho(y)}} \cdot [1 - \phi(x)\overline{\phi(y)}]^{-1}$$

For this space and a fixed positive integer n Richter-Dyn considers quadrature formulas of the type $\sum_{i=1}^{n} a_i f(y_i)$, where a_i, y_i are real and y_i

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