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CELL-LIKE MAPPINGS OF HILBERT CUBE MANIFOLDS: APPLICATIONS TO SIMPLE HOMOTOPY THEORY

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ABSTRACT. In this note an infinite-dimensional result is established which implies the following finite-dimensional result as a special case: If K, L are finite CW-complexes and f is a map of K onto L such that each point-inverse has trivial shape, then f is a simple homotopy equivalence.

1. **Introduction.** A *Hilbert cube manifold*, or *Q-manifold*, is a separable metric manifold modeled on the Hilbert cube Q. A mapping $f: X \to Y$ is said to be *CE*, or *cell-like*, provided that f is onto, proper (i.e. the inverse image of each compactum is compact), and each point-inverse $f^{-1}(y)$ has trivial shape (in the sense of Borsuk [**1**]). Here is the main result of this note.

THEOREM 1. *If X, Y are Q-manifolds and $f: X \to Y$ is a CE mapping, then f is proper homotopic to a homeomorphism of X onto Y.*

The key technical result needed for the proof of Theorem 1 is the solution of an infinite-dimensional *CE* handle problem, which is stated in Lemma 2 here and is the main result of [**7**]. The proof of Lemma 2 uses a considerable amount of infinite-dimensional topology along with the torus technique of [**10**], which was crucial in establishing a corresponding finite-dimensional result.

A CW-complex is *strongly locally-finite* provided that it is the union of a countable, locally-finite collection of finite subcomplexes. The following is an application of Theorem 1 to infinite simple homotopy equivalences of strongly locally-finite CW-complexes (see [**9**] for a definition of an infinite simple homotopy equivalence).

THEOREM 2. *If K, L are strongly locally-finite* CW*-complexes and f: $K \to L$ is a CE mapping, then f is an infinite simple homotopy equivalence.*

This generalizes a result of the author's [**6**], where it was shown that any homeomorphism between strongly locally-finite CW-complexes is an infinite simple homotopy equivalence. We remark that Cohen [**8**] had

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