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REPRESENTATIONS OF GENERALIZED MULTIPLIERS OF L^p-SPACES OF LOCALLY COMPACT GROUPS

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The objective of this note is to announce a class of generalized multipliers between L^p-spaces of locally compact groups and some characterizations obtained by the author which generalize the classical representations of Figà Talamanca, Gaudry, Rieffel and others. If G is a locally compact group, let $L^p(G)$, $1 \leqq p \leqq \infty$, denote the corresponding Lebesgue spaces relative to a fixed Haar measure dx (with the convention that dx is normalized if G is compact). Let L_x for $x \in G$ denote the left translation operator on $L^p(G)$ given by $L_x f(y) = f(x^{-1}y)$. Let G, H, and K be locally compact groups and let $\theta: K \to G$ and $\psi: K \to H$ be continuous group homomorphisms. Let $1 \leqq p, q \leqq \infty$. We define a *$(\theta, p; \psi, q)$-multiplier* to be a bounded linear transformation $T: L^p(G) \to L^q(H)$ such that $T \circ L_{\theta(z)} = L_{\psi(z)} \circ T$ for all $z \in K$. Let $\mathrm{Hom}_K(L^p(G), L^q(H))$ denote the Banach space with the operator norm of all $(\theta, p; \psi, q)$-multipliers of $L^p(G)$ into $L^q(H)$. When $G = H = K$ and $\theta = \psi = \mathrm{id}_G$ (the identity map on G) then a $(\mathrm{id}_G, p; \mathrm{id}_G, q)$-multiplier is a "classical" (p, q)-multiplier of $L^p(G)$ into $L^q(G)$.

In [**1**] and [**2**], Figà-Talamanca and Gaudry have shown the "classical" multiplier space $\mathrm{Hom}_G(L^p(G), L^q(G))$ is isometrically isomorphic to the Banach space dual of the Banach space $A_p^{q'}(G)$ [**14**, Definitions 3.2 and 5.4] of functions on G for LCA groups G where $1/q + 1/q' = 1$. Rieffel [**14**] has extended this representation to amenable locally compact groups (using an approximation theorem of C. S. Herz when G is possibly noncompact). The representation for general G is still an open problem.

In this note we describe extensions of the above cited representations to the space of $(\theta, p; \psi, q)$-multipliers. Our approach parallels that of Rieffel in [**14**] by using tensor products of Banach modules. We assume familiarity with the general results concerning tensor products of Banach modules in [**13**]; specifically, if V and W are left and right Banach A-modules for a Banach algebra A then by [**13**, Corollary 2.13]

$$(1.1) \qquad (V \otimes_A W)^* \cong \mathrm{Hom}_A(V, W^*),$$

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