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ASYMPTOTIC THEOREMS FOR SUMS OF INDEPENDENT RANDOM VARIABLES DEFINED ON A TREE[1]

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The study of sums of independent random variables defined on a tree has not been treated systematically in the literature, except for the random tree generated by a Galton-Watson process (cf. [1], [4], [5]) and for the binary tree (cf. [3]). The purpose of this short note is to announce a generalization of the results of the above papers.

1. A tree $\mathscr{T}$ will be here a collection of sequences $\tau = (i_1 \cdots i_k \cdots)$ where the i_j are nonnegative integers such that

(a) if $i_l = 0$, then $i_k = 0$ for all $k > l$.

(b) $i_1 = 1 \cdots Z_1$.

(c) for $k > 1$, $i_k = 1 \cdots Z_{i_1 \cdots i_{k-1}}$ or 0, $\sum_{i_1 \cdots i_{k-1}} Z_{i_1 \cdots i_{k-1}} = Z_k$. We require $Z_k \geqq 1$.

Given a tree $\mathscr{T}$, we define $\mathscr{T}_k$, the family of size k of $\mathscr{T}$, to be the set of finite sequences $\tau_k = (i_1 \cdots i_k)$ of length k which are the beginning of a sequence of the tree such that $i_k \neq 0$. The cardinality of $\mathscr{T}_k$ is Z_k. We denote by $\alpha(n, k)$ the number of ordered pairs of the path of $\mathscr{T}_n$ which have exactly in common an initial path of length k. Let $p_{n,k} = \alpha(n, k)/Z_n^2$; we say that the tree is regular if $\lim_{n\to\infty} p_{n,k} = p_k$ exists with $\sum_k p_k = 1$. Let g be a nonnegative nondecreasing function defined on the integers.

(a) We say that the tree $\mathscr{T}$ is g-regular if it is regular and if

$$\lim_{n\to\infty} \sum_{k=0}^{n} g(k)p(n, k) = \sum_{k=0}^{\infty} g(k)p_k < \infty.$$

(b) We say that the tree $\mathscr{T}$ is weakly g-regular if

$$\sup_n \sum_{k=0}^{n} g(k)p(n, k) < \infty.$$

We consider a family of independent identically distributed random variables X_{τ_k} indexed by $\bigcup_{k=1}^{\infty} \mathscr{T}_k$. To simplify notations and the statement of the theorems we assume the X's to have mean 0 and variance 1. At each path $\tau_n = (i_1 \cdots i_n)$ we associate the random variables

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