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SOME RESULTS IN E. CARTAN'S THEORY OF ISOPARAMETRIC FAMILIES OF HYPERSURFACES[1]

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In a series of papers [1]–[4], E. Cartan developed the theory of isoparametric families of hypersurfaces and proposed a number of problems. The purpose of the present note is to announce the following three results. First, we construct a series of isoparametric families of hypersurfaces M_t^{2n} in S^{2n+1}, $n \geqq 2$, thus providing an affirmative answer to one of Cartan's problems. Second, we show that each focal variety belonging to an isoparametric family M_t^n in S^{n+1} admits a global submanifold structure if M_t^n consists of compact hypersurfaces. Finally, we prove that each focal variety of M_t^n is a minimal submanifold in S^{n+1}.

In §1, we recall, very briefly, some basic facts from Cartan's work. In §2, we give the construction of isoparametric families M_t^{2n} in S^{2n+1}. In §3 we discuss focal varieties, in particular, from a global point of view. In §4 we deal with minimality of focal varieties. The details will appear elsewhere together with a systematic account of Cartan's theory of isoparametric families.

1. **Isoparametric family of hypersurfaces.** A connected hypersurface M^n in the sphere S^{n+1} (unit hypersphere in Euclidean space R^{n+2}) is said to have constant principal curvatures if there are distinct constants $a_1, \ldots, a_p$ which, for a suitable choice of a unit normal vector field ξ, represent all the distinct principal curvatures at every point. In this case, the multiplicity v_i of each a_i remains the same throughout M. Of course, $n = \sum_{i=1}^p v_i$. Let M_t^n be a family of parallel hypersurfaces obtained by moving each point of M^n by distance t along the geodesic in the direction

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