BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 79, Number 6, November 1973

ON THE ARITHMETIC OF TUBE DOMAINS (BLOWING-UP OF THE POINT AT INFINITY)

BY I. SATAKE

The purpose of this talk is to construct, in a certain canonical manner, a "blow-up" of the isolated singularity which appears as a point at infinity of an arithmetic quotient of a symmetric tube domain. Similar (but slightly different) blowing-ups have already appeared in some special cases in the works of Pyatetskii-Shapiro, Igusa [4] (Siegel modular case) and Hirzebruch [3] (Hilbert modular case). It should be possible to extend our construction to the case of general symmetric domains via their realizations as "Siegel domains of the third kind" (cf. [1], [6], [7b]). But, for the sake of simplicity, we shall here restrict ourselves to the simplest case.[1]

1. Let U be an n-dimensional real vector space endowed with a (positive-definite) inner product $\langle\ \ \rangle$ and Ω a (nonempty) open convex cone in U with its vertex at the origin of U. We assume that Ω does not contain any straight line (not necessarily passing through the origin). Let G_0 be the identity connected component of the (linear) automorphism group

$$\mathrm{Aut}(\Omega) = \{g \in \mathrm{GL}(U) \mid g\Omega = \Omega\}.$$

In the following, we assume Ω to be "homogeneous" and "self-dual"; these mean that G_0 is transitive on Ω and that Ω coincides with its dual

$$\Omega^* = \{u \in U \mid \langle u, u'\rangle > 0 \text{ for all } u' \in \overline{\Omega} - \{0\}\},$$

where $\overline{\Omega}$ denotes the closure of Ω. It is known [9] that the latter condition is equivalent to saying that ${}^tG_0 = G_0$, t denoting the adjoint with respect to the inner product $\langle\ \ \rangle$, and this implies that G_0 is the identity connected

An expanded version of an invited address delivered at the 698th meeting of the Society in La Jolla, California on November 18, 1972; received by the editors May 7, 1973. I wish to express here my thanks to Mumford, Hartshorne, and Yamaguchi for very helpful comments on my original manuscript.

AMS (MOS) subject classifications (1970). Primary 32N15, 10E99, 14M20.

Key words and phrases. Self-dual cones, arithmetic groups, analytic spaces.

[1] After the talk I have learned, by a written communication with Mumford, that a closely related theory has been developed by Demazure, Mumford and others; compare his recent seminar notes at Harvard [5]; cf. also M. Demazure, *Sous-groupes algebriques de rang maximum du groupe de Cremona*, Ann. Sci. Ecole Norm. Sup **3** (1970), and M. Hochster, *Rings of invariants of tori, Cohen-Macaulay rings generated by monomials and polytopes*, Ann. of Math. **96** (1972). It seems that a fair part of our results overlaps with theirs obtained in a much wider framework. Some connections with their theory are alluded to in §4 and §9 of the text.