BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 79, Number 5, September 1973

SECTIONAL CURVATURE IN PIECEWISE LINEAR MANIFOLDS

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Communicated by William Browder, February 13, 1973

A *metric complex* M is a connected, locally-finite simplicial complex linearly embedded in some Euclidean space R^l. Metric complexes M and M' are *isometric* if they have subdivisions L and L' and if there is a simplicial isomorphism $h:L \to L'$ such that for every $a \in L$, the linear map determined by $h:a \to h(a)$ is an isometry (that is, it extends to an isometry of the affine spaces generated by these simplexes). This note is concerned with certain characteristics of a metric complex M which are *intrinsic*, i.e., which depend only on the isometry class of M. The basic such characteristic is the *intrinsic metric*, which is best described in the piecewise linear context by H. Gluck [**3**]; for a more general treatment see W. Rinow [**8**].

Let $M \subseteq R^l$ be a metric complex and let p be a point of M. Then the *tangent cone* T_pM of M at p is defined to be the infinite cone with vertex p generated by link(p, M). The isometry class of T_pM is intrinsic to M, for each p. An infinite ray $p\bar{x}$ in T_pM will be called a *tangent direction* at p to M.

Let N_pM be a subcone of T_pM and let j be a nonnegative integer. Let $R^j \times N_pM$ be given the metric in which its factors are orthogonal. For various choices of N_pM and j, $R^j \times N_pM$ will be isometric to T_pM. For example if p is in the interior of a j-simplex of M, such a factoring exists. Consider those factorings of T_pM for which j is maximal; then the corresponding N_pM are all isometric. Such an N_pM will be called the *normal geometry* of p in M, and denoted ν_pM. For example, if M is an n-manifold and p is in the interior of an $(n-1)$- or n-simplex, then $\nu_pM = \{p\}$. If M is a 2-manifold, then $\nu_pM = \{p\}$ unless p is a vertex of nonzero curvature, when $\nu_pM = T_pM$.

Clearly j and ν_pM determine the metric geometry of M near p.

For any $p \in M$ and any tangent direction $p\bar{x}$ at p lying in ν_pM I have defined numbers $k_+(p\bar{x})$ and $k_-(p\bar{x})$, with $k_+(p\bar{x}) \geqq k_-(p\bar{x})$, called the *maximum* and *minimum curvatures* of M at p in the direction $p\bar{x}$. The definitions are too long to give here. Roughly speaking, $k_-(p\bar{x})$ equals: 2π minus twice the maximum "angle" that can occur between $p\bar{x}$ and any other $p\bar{y} \subseteq \nu_pM$ as y varies; $k_+(p\bar{x})$ is defined similarly, using a

AMS (MOS) subject classifications (1970). Primary 53C70, 57C99; Secondary 53C45.
[1] Research supported in part by NSF Grants P22927 and P029431000.