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TOPOLOGICAL SCHUR LEMMA AND RELATED RESULTS

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We announce here some results of a paper to appear elsewhere [**1**].

Let a torus T act continuously on a topological space X. Let $X \to X_T \to^{\pi} B_T$ be the fibre bundle with fibre X associated (by means of the action of T on X) to the universal principal T bundle $T \to E_T \to B_T$. We define the equivariant cohomology ring $H_T^*(X) = H^*(X_T)$ where H^* denotes Čech cohomology with rational coefficients. When Y is an invariant subspace of X, we define $H_T^*(X, Y) = H^*(X_T, Y_T)$. Then $R = H^*(B_T)$ is a polynomial ring and $H_T^*(X, Y)$ is a module over R by means of π^*.

For each subtorus L of T let PL be the kernel of $H^*(B_T) \to H^*(B_L)$. Let $X^L = F(L, X)$ be the set of points fixed by L. We will assume that X is compact. Given a closed invariant subspace $Y \subset X$ and an element $x \in H_T^*(Y)$, we define

$$I_x = \{a \in R \mid ax \text{ lies in the image of } H_T^*(X) \to H_T^*(Y)\}, \quad \text{and}$$

$$I_x^L = \{a \in R \mid ax \text{ lies in the image of } H_T^*(X^L \cup Y) \to H_T^*(Y)\}.$$

When $L \subset K$ are subtori, $I_x \subset I_x^L \subset I_x^K$. We say that K *belongs to* x if K is maximal with respect to the property $I_x^K \neq R$.

1. THEOREM. *The isolated primary components of the ideal I_x are the ideals I_x^K where K belongs to x. The radical of I_x^K is PK, hence $\sqrt{I_x} = \bigcap PK$ where K ranges over the subtori belonging to x.*

2. COROLLARY. *If I_x is principal, the subtori belonging to x are all of corank* 1 *and $I_x = \bigcap I_x^K$ where K ranges over the subtori belonging to x. For each such K, $I_x^K = (\omega^d)$ where $d \geqq 1$ and $\omega \in H^2(B_T)$ generates PK.*

Assume that the fixed point set F of the T action on X is not connected. Let $F = F^1 + \cdots + F^s$ be the connected components of the fixed point set, $s \geqq 2$. We say that a subtorus L connects F^1 and F^2 if they lie in the same component of X^L. We assume that dim $H^*(X)$ is finite.

3. THEOREM. *Let $N \subset H_T^*(X)$ be the ideal generated by odd degree and R torsion elements. Assume that $H_T^*(X)/N$ is generated by k elements as an R algebra. Then for every maximal subtorus K connecting F^1 and F^2,* rank $K \geqq$ rank $T - k$.

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