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A THEORY OF PROPER SHAPE FOR LOCALLY COMPACT METRIC SPACES

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The notion of topological shape, first introduced by K. Borsuk ([**1**], [**2**]) for compact metric spaces, has been extended to larger classes of spaces in a number of ways ([**3**], [**4**], [**10**], [**6**], [**8**], [**11**]). Not all of these extensions agree, but all retain the property of the original formulation that any two spaces of the same homotopy type have the same shape. Any extension to noncompact spaces which retains this property, however, must necessarily sacrifice much of the geometric flavor of the original definition, since homotopy type is notoriously ungeometric in the noncompact case. For noncompact spaces which are locally nice (e.g., locally compact ANR's), *proper* homotopy type gives a much more geometric classification. Since the proper homotopy type of a compactum (in the class of compacta) is the same as its homotopy type, Borsuk's original concept of shape could, perhaps, more appropriately be considered a generalization of proper homotopy type than of homotopy type, and extended to noncompact spaces so as to retain this property.

Of several natural extensions which apply to classes of spaces including noncompact ones and which generalize proper homotopy type, we have chosen to present one which, although limited to locally compact metrizable spaces, seems to emphasize geometric properties to a greater extent than do some alternatives which apply to larger classes of spaces. Moreover, this particular treatment very closely parallels Borsuk's original development, and in many instances virtually the same proofs apply. A detailed version, along with proofs of the results claimed here, will be published at a later date.

In order to avoid difficulties arising from the fact that closed sets in noncompact spaces may not have countable neighborhood bases, we substitute "fundamental nets" $\{f_\lambda \mid \lambda \in \Lambda\}$, Λ a directed set, for the fundamental sequences $\{f_k \mid k = 1, 2, \ldots\}$ of Borsuk. And in order that our extension should generalize proper homotopy type rather than homotopy type, we require that all homotopies involved in the definitions should

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