BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 79, Number 5, September 1973

DUALITY IN CROSSED PRODUCTS AND VON NEUMANN ALGEBRAS OF TYPE III

BY MASAMICHI TAKESAKI

Communicated by Jacob Feldman, December 1, 1972

In this paper, we announce further results succeeding to the previous papers [2] and [3]. Suppose $\mathscr{M}$ is a von Neumann algebra and G an abelian locally compact group. Let $\sigma : t \in G \to \sigma_t \in \mathrm{Aut}(\mathscr{M})$ be a continuous homomorphism in the sense that for each $x \in \mathscr{M}$, the map: $t \in \boldsymbol{R} \to \sigma_t(x) \in \mathscr{M}$ is σ-weakly continuous. We construct the crossed product $\mathscr{M} \otimes_\sigma G$ of $\mathscr{M}$ by G with respect to σ. In [2], we have shown that there is a canonical dual action θ of the dual group $\hat{G}$ on $\mathscr{M} \otimes_\sigma G$ so that $(\mathscr{M} \otimes_\sigma G) \otimes_\theta \hat{G} \cong \mathscr{M} \otimes \mathscr{L}(L^2(G))$, where $\mathscr{L}(L^2(G))$ means, of course, the algebra of all bounded operators on the Hilbert space $L^2(G)$ of all square integrable functions on G with respect to the Haar measure of G.

THEOREM 1. (i) *If H is a closed subgroup of G, then $\mathscr{M} \otimes_\sigma H$ is canonically imbedded in $\mathscr{M} \otimes_\sigma G$ and $\{p \in \hat{G} : \theta_p(x) = x$ for every $x \in \mathscr{M} \otimes_\sigma H\}$ is precisely the annihilator $H^\perp$ of H.*

(ii) *If $\hat{H}$ is a closed subgroup of $\hat{G}$, then the fixed point subalgebra $(\mathscr{M} \otimes_\sigma G)^{\hat{H}}$ of $\mathscr{M} \otimes_\sigma G$ under $\theta(\hat{H})$ is precisely $\mathscr{M} \otimes_\sigma H$, with H the annihilator $\hat{H}^\perp$ of $\hat{H}$ in G.*

We apply this theorem to the structure of von Neumann algebras of type III. In [3], we showed that for a von Neumann algebra $\mathscr{M}$ of type III, there exists uniquely a semifinite von Neumann algebra $\mathscr{M}_0$ equipped with a one parameter automorphism group $\{\theta_t\}$ such that $\mathscr{M} \cong \mathscr{M}_0 \otimes_\theta \boldsymbol{R}$ and the action σ_t of $\boldsymbol{R}$ on $\mathscr{M}$, which is dual to θ, is the modular automorphism group associated with the faithful semifinite normal weight φ which is canonically constructed from a trace τ on $\mathscr{M}_0$ with $\tau \cdot \theta_t = e^t\tau$, $t \in \boldsymbol{R}$. The above theorem implies immediately the following result.

COROLLARY 2. *Imbedding canonically $\mathscr{M}_0$ into $\mathscr{M} = \mathscr{M}_0 \otimes_\theta \boldsymbol{R}$, $\mathscr{M}_0$ is precisely the centralizer of the weight φ.*

COROLLARY 3. *The center $\mathscr{Z}$ of $\mathscr{M}$ is precisely the fixedpoint subalgebra of the center $\mathscr{Z}_0$ of $\mathscr{M}_0$ under the action $\{\theta_t : t \in \boldsymbol{R}\}$.*

We now consider a factor $\mathscr{M}$ of type III with separable predual $\mathscr{M}_*$. Making use of measure theoretic arguments, which are partly due to

---

AMS (MOS) subject classifications (1970). Primary 46L10.