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EXTENSIONS OF C^*-ALGEBRAS, OPERATORS WITH COMPACT SELF-COMMUTATORS, AND K-HOMOLOGY[1]

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1. **Introduction.** The study of a certain class of extensions of C^*-algebras is suggested by recent developments in two diverse areas of mathematics. Starting from the classical results of Weyl and von Neumann on compact perturbations of selfadjoint operators, operator theorists have become increasingly interested in operators which are normal modulo the compacts. Such operators generate extensions of the desired type and the study of these operators can be reduced to that of the corresponding extensions. Along these lines our results on extensions yield that an operator with compact self-commutator and essential spectrum not separating the plane is a normal plus a compact, and that the set of the normals plus the compacts is norm-closed.

Further, the algebras generated by pseudo-differential operators on manifolds also define such extensions as do the algebras generated by certain collections of Toeplitz and Wiener-Hopf operators. Moreover, these extensions can be used to realize "concretely" a K-homology theory outlined by Atiyah [**1**]. It seems likely that this functor will have applications in index theory and topology.

Although unanswered questions (which we describe later) remain, our results are reasonably complete, especially in regard to lower dimensional spaces which are of principal interest in the applications to operator theory. Definitions and some results appear in [**2**]. Proofs of the remaining results will appear later.

2. **The functor** Ext. Let $\mathscr{H}$ be a separable infinite dimensional complex Hilbert space, $\mathscr{L}(\mathscr{H})$ the algebra of bounded operators on $\mathscr{H}$, $\mathscr{K}$ the ideal of compact operators, $\mathfrak{A}$ the quotient (Calkin) algebra $\mathscr{L}(\mathscr{H})/\mathscr{K}$ and $\pi:\mathscr{L}(\mathscr{H}) \to \mathfrak{A}$ the quotient map. Let X be a compact metrizable space and $C(X)$ the C^*-algebra of continuous complex functions on X. We are interested in the C^*-algebra extensions of $\mathscr{K}$ by $C(X)$. We require that the algebras have identity and a further condition which guarantees

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