BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 79, Number 5, September 1973

ENERGY DECAYS LOCALLY EVEN IF TOTAL ENERGY GROWS ALGEBRAICALLY WITH TIME

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Communicated by Cathleen Morawetz, February 13, 1973

0. **Introduction.** In this note we announce energy decays locally like $t^{-2+\kappa}$ for solutions of hyperbolic equations, with coefficients that depend upon both position and time, in the exterior of star-shaped domains in $\boldsymbol{R}^3$. Here κ is a positive constant, depending on the coefficients, defined explicitly by (8) below. Our results generalize those of Zachmanoglou [**4**]. He considered a class of equations with time-independent coefficients (see (12) below) and proved under hypotheses roughly analogous to ours that in $\boldsymbol{R}^n$ ($n \geqq 3$) energy decays locally like $t^{-1+\mu}$ ($1 > \mu \geqq 0$). A more important difference between the equations we consider here and those considered by Zachmanoglou in [**4**] is that we treat equations with solutions whose total energy may grow algebraically with t while the total energy of solutions of the equations considered in [**4**] is conserved. In [**1**] we proved that the energy of solutions with bounded total energy decays locally like t^{-2}, but under more stringent hypotheses than those used here.

We now set the scattering problem whose solutions we investigate. Let V be the exterior of a closed, bounded subset B of $\boldsymbol{R}^3$, and let n be the outward unit normal to ∂B. We assume that the origin lies interior to B and that $\partial V \equiv \partial B$ is star-shaped:

$$\min_{x\in\partial V} \frac{n \cdot x}{r} \geqq 0, \tag{1}$$

where $x = (x_1, x_2, x_3)$ and $r^2 = x \cdot x$. Let $Q = (V \cup \partial V) \times [0, \infty)$. We use the notation $\nabla^{(3)} = (\partial/\partial x_1, \partial/\partial x_2, \partial/\partial x_3)$, $\nabla = (\nabla^{(3)}, \partial/\partial t)$. We take as given a symmetric 3×3 matrix E, 1×3 matrices a and b, and functions c and d which satisfy the following hypothesis:

(Hypothesis H_1) (a) *b, c, and E are in $C^1(Q)$; a and d are in $C^2(Q)$,*
(b) *for some $d_0 > 0$, $d(x, t) \geqq d_0$ if $(x, t) \in Q$.*

Let the transpose of a matrix M (or m) be M^T (or m^T). We suppose that E is uniformly elliptic in Q, namely that there exist positive constants c_0 and

AMS (MOS) subject classifications (1970). Primary 35B05, 35L10, 78A45; Secondary 35P25, 78A05.

Key words and phrases. Energy decay, star-shaped obstacle, second order hyperbolic equations, variable coefficients, divergence identity.