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SPLITTING OBSTRUCTIONS FOR HERMITIAN FORMS AND MANIFOLDS WITH $Z_2 \subset \pi_1$

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Theorem 1 of this announcement constructs explicit algebraic counterexamples to the general conjecture (see for example [**W**, p. 138]) that groups of Hermitian forms satisfy a sum formula for free products. This conjecture was verified when the relevant groups have no 2-torsion by proving equivalent codimension one splitting theorems for manifolds. In part II of this note, see especially Theorem 7, the failure of the general algebraic conjecture leads to examples of nonsplittable manifolds with $Z_2 \subset \pi_1$. Some of the geometric splitting obstructions occur as differences of Arf-Kervaire invariants of base spaces and covering spaces.

I. Let (G, ω) be a group G equipped with a homomorphism $\omega: G \to Z_2$. $L_n(G, \omega)$ denotes the Wall surgery obstruction group to the simple homotopy equivalence problem for manifolds with fundamental group G and orientation homomorphism ω [**W**]. For $n = 2k$ these are Grothendieck groups of $(-1)^k$ Hermitian forms over the integral group ring $Z[G]$; for n odd, these are abelian quotients of unitary groups over $Z[G]$. When ω is trivial, write simply $L_n(G) = L_n(G, \omega)$, and for the reduced group write $\tilde{L}_n(G)$, where $L_n(G) = \tilde{L}_n(G) \oplus L_n(0)$. Write Z (resp. Z_2) for the integers (modulo 2).

The conjecture referred to above is that $\tilde{L}_n(G_1 * G_2) = \tilde{L}_n(G_1) \oplus \tilde{L}_n(G_2)$. For G_1 and G_2 finitely presented and *without* elements of order 2, this was proved first by R. Lee for n even [**L**], and for all n by the author as a special case of a general result on surgery groups of amalgamated free products [**C1**] [**C2**] [**C3**]. For $n = 4k$, the author proved the above conjecture for G_1 and G_2 finitely presented groups [**C2**] [**C3**]. However, Theorem 1 limits the possible further extensions of this. Note that for $G = Z$ Theorem 1 provides a counterexample to a theorem of [**M**, p. 676].

THEOREM 1. *Let G be a nontrivial cyclic group. Then*

$$Z_2 \subset L_{4k+2}(Z_2 * G)/L_{4k+2}(Z_2) + L_{4k+2}(G).$$

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