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RESEARCH ANNOUNCEMENTS

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CLASSIFYING RELATIVE EQUILIBRIA. I

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Introduction. We announce several theorems which place us nearer the goal of classifying relative equilibria in the planar n-body problem. These theorems answer some of the questions on the nature of relative equilibria which were raised recently by S. Smale [**3**], [**4**]. We refer to these papers for definitions left unspecified here. It is a pleasure to acknowledge the encouragement of S. Smale in these pursuits.

1. **Relative equilibria defined.** We study a real analytic function $\tilde{V}_m$ on a real analytic manifold X_m where $n \geqq 3$ and $m = (m_1, \ldots, m_n) \in R^n_+$ are fixed. For each n, m, X_m is homeomorphic to $P_{n-2}(C) - \tilde{\Delta}_{n-2}$, an open manifold of dimension $2n - 4$. $\tilde{\Delta}_{n-2}$ is the union of $n(n-1)/2$ codimension 1 complex projective subspaces. $\tilde{V}_m$ is the potential function which is induced on X_m by V_m, the potential function of the planar n-body problem.

The classical definition of relative equilibria [**5**, p. 286] and the modern one [**4**, p. 47] are equivalent. For our purposes we may consider a class of relative equilibria to be a critical point $x \in X_m$ of $\tilde{V}_m$. It is known that $\tilde{V}_m$ is a proper map and from [**2**] that the critical points of $\tilde{V}_m$ are bounded away from the fat diagonal $\tilde{\Delta}_{n-2}$. Therefore, provided $\tilde{V}_m$ is nondegenerate, we can apply Morse theory in order to count critical points.

As a first step we need to know the homology of the manifold $P_{n-2}(C) - \tilde{\Delta}_{n-2}$ before we attempt to apply the Morse inequalities directly. The answer as given by Theorem 1 is a recurrence relation for the

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