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STATISTICAL GEOMETRY: A TOOL FOR PATTERN ANALYSIS[1]

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0. **Summary.** This paper deals with the analysis and recognition of two-dimensional set patterns deformed by certain deformation mechanisms. The emphasis is on exploiting the form of the image algebra and the behavior of the deformations. In this way higher recognition efficiency is obtained by using structure-preserving restoration.

1. **The structure of patterns.** Patterns can be described through the following general formalism. They are generated from primitives called signs and denoted by s, $s \in \mathscr{S}$. Finite vectors $c = (c_1, c_2, \ldots, c_m) \in \mathscr{C}$ are called configurations and in the set $\mathscr{C}$ of all legal configurations an equivalence relation R gives rise to equivalence classes I called images. A set of rules $\mathscr{R}$ determines what configurations are legal. The set $\mathscr{T}$ of resulting images is called the image algebra. Expressing configurations just as vectors is sufficient when we have just one binary operation between signs. If several binary operations are used a more expressive notation is needed; see below. The images I are the objects that can be observed (under ideal conditions) and, depending upon the way they have been generated, they are grouped into pattern classes $\mathscr{P}_\alpha$ forming a family $\mathscr{P}$ of patterns. The reader is referred to Grenander (1970) for more details.

Under actual conditions the images cannot be observed exactly. Instead, a deformation mechanism $\mathscr{D}$ maps the set $\mathscr{T}$ of pure images into a set $\mathscr{T}^{\mathscr{D}}$ of deformed images. *The purpose of pattern analysis is to describe the generation of $\mathscr{T}$, the mapping into $\mathscr{T}^{\mathscr{D}}$, and to design algorithms for the analysis and recognition of I given $I^{\mathscr{D}}$ or, at least, the partial restoration of I.* Finally this will be used for classification into the pattern classes $\mathscr{P}_\alpha$.

The main difficulty is usually caused by the way that $\mathscr{D}$ obscures the view of the pure image I. Often the deformation mechanism $\mathscr{D}$ is not even discussed explicitly, but only assumed vaguely when the recognition algorithm is suggested. In this paper we shall show how to take $\mathscr{D}$ into account for the special case of two-dimensional set patterns and for a

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