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SKEW-PRODUCT FLOWS, FINITE EXTENSIONS OF MINIMAL TRANSFORMATION GROUPS AND ALMOST PERIODIC DIFFERENTIAL EQUATIONS[1]

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I. **Skew-product flows.** A flow π on a product space $X \times Y$ is said to be a *skew-product flow* if there exist continuous mappings $\varphi : X \times Y \times T \to X$ and $\sigma : Y \times T \to Y$ such that

$$\pi(x, y, t) = (\varphi(x, y, t), \sigma(y, t))$$

where σ is itself a flow on Y and T is a topological group. In other words the natural projection $p : X \times Y \to Y$ is a homomorphism of the transformation group $(X \times Y, T, \pi)$ onto (Y, T, σ).

Skew-product flows arise in a natural way in the study of ordinary differential equations $x' = g(x, t)$ (cf. [**6**] and [**7**]). In this case the group T would be the real numbers and Y would be a topological function space containing g and closed under time-translations. The flow σ would be given by $\sigma(f, \tau) = f_\tau$ where $f_\tau(x, t) = f(x, \tau + t)$. The space X would be the phase space for the differential equation, usually X is the Euclidean space R^n or perhaps some n-dimensional manifold, and $\varphi(x, f, t)$ would represent the solution of $x' = f(x, t)$ passing through x at time $t = 0$. (We assume that all differential equations in Y give rise to unique solutions, although some of our results are valid without this restriction (cf. [**8**]).)

Now assume that Y is a compact minimal set under the flow σ and let $M \subset X \times Y$ be a compact invariant set of the skew-product flow. Motivated by the above model for differential equations we ask: When can certain structures be lifted from Y to M? For example, if we assume that Y is an almost periodic minimal set (that is, the flow σ is equicontinuous on Y) under what conditions will M contain an almost periodic minimal set?

We shall say that the flow π has the *distal property* on M if for any $y \in Y$ and $x_1, x_2 \in X$ with $x_1 \neq x_2$, $(x_1, y) \in M$ and $(x_2, y) \in M$ there is an

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