COHOMOLOGY OF BRAID SPACES

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1. Introduction and results. Let M be a manifold; define F(M, k) to be the subspace $\{\langle x_1, \dots, x_k \rangle | x_i \in M, x_i \neq x_j \text{ if } i \neq j \}$ of M^k . There is a proper right action of Σ_k , the symmetric group on k-letters, on F(M, k)given by

$$\sigma \cdot \langle x_1, \ldots, x_k \rangle = \langle x_{\sigma(1)}, \ldots, x_{\sigma(k)} \rangle, \qquad \sigma \in \Sigma_k.$$

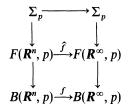
Let B(M, k) denote the orbit space $F(M, k)/\Sigma_k$. The object of this paper is to outline the calculation of

$$H^*(\operatorname{Hom}_{\Sigma_p}(C_*F(\mathbf{R}^n, p); \mathbf{Z}_p(q))), \qquad n \ge 2, p \text{ prime},$$

where $C_*F(\mathbf{R}^n,p)$ denotes the singular chains of $F(\mathbf{R}^n,p)$, and $\mathbf{Z}_p(q)$ denotes the Σ_p -module \mathbf{Z}_p with Σ_p -action $\sigma \cdot x = (-1)^{qs(\sigma)}x$ for $x \in \mathbf{Z}_p$ and $\sigma \in \Sigma_p ((-1)^{s(\sigma)})$ is the sign of σ). Since the Σ_p -action on $F(\mathbb{R}^n, p)$ is proper, we may identify $H^*(\operatorname{Hom}_{\Sigma_n}(C_*F(\mathbf{R}^n, p); \mathbf{Z}_p(2q)))$ with $H^*(B(\mathbf{R}^n, p); \mathbf{Z}_p)$ [5]. By abuse of notation we denote $H^*(\operatorname{Hom}_{\Sigma_p}(C_*F(\mathbb{R}^n,p);\mathbb{Z}_p(q)))$ by $H^*(B(\mathbf{R}^n, p); \mathbf{Z}_p(q)).$

The interest in $H^*(B(\mathbf{R}^n, p); \mathbf{Z}_p(q))$ arises from the work of Peter May [6], [7] which implies that each class in $H_*(B(\mathbb{R}^n, p); \mathbb{Z}_p(q))$ determines a homology operation on all classes of degree q in the mod p homology of any n-fold loop space.

For our calculations, we rely heavily on the map of fibrations



where $F(\mathbf{R}^{\infty}, p) = \varinjlim F(\mathbf{R}^{n}, p)$ and $B(\mathbf{R}^{\infty}, p) = F(\mathbf{R}^{\infty}, p)/\Sigma_{p}$. Here f and \widehat{f} are the evident inclusions. Since $F(\mathbf{R}^{\infty}, p)$ is contractible with free Σ_p -action, $B(\mathbf{R}^{\infty}, p)$ is a $K(\Sigma_p, 1)$. Obviously

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