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DEGREE THEORY FOR NONCOMPACT MULTIVALUED VECTOR FIELDS

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Introduction. In this note we indicate the development and state the properties of a degree theory for a rather general class of multivalued mappings, the so-called ultimately compact vector fields, and then use this degree to obtain fixed point theorems. As will be seen, these results unite and extend the degree theory for single-valued ultimately compact vector fields in [**13**] and the degree theory for multivalued compact vector fields in ([**5**], [**8**]) and also serve to extend to multivalued mappings the fixed point theorems for single-valued mappings obtained in [**1**], [**2**], [**3**], [**9**], [**10**], [**13**], and others (see [**13**]) and to more general multivalued mappings the fixed point theorems in [**4**], [**6**], [**8**]. The detailed proofs of the results presented in this note will be published elsewhere.

1. Let X be a metrizable locally convex topological vector space. If $D \subset X$ we denote by $K(D)$ and $CK(D)$ the family of closed convex, and the family of compact convex subsets of D, respectively. We also use $\bar{D}$ (or cl D), ∂D, and clco D to denote the closure, boundary and convex closure of D, respectively. To define what we mean when we say that the upper semicontinuous (u.s.c.) mapping $T: D \to K(X)$ is ultimately compact, we employ a construction of a certain transfinite sequence $\{K_\alpha\}$ utilized by Sadovsky [**13**] in his development of the index theory for ultimately compact single-valued vector fields. Let $K_0 = \operatorname{clco} T(D)$, where $T(A) = \bigcup_{x \in A} T(x)$ for $A \subset D$. Let η be an ordinal such that K_β is defined for $\beta < \eta$. If η is of the first kind we let $K_\eta = \operatorname{clco} T(D \cap K_{\eta-1})$, and if η is of the second kind we let $K_\eta = \bigcap_{\beta < \eta} K_\beta$. Then $\langle K_\alpha \rangle$ is well defined and such that $K_\alpha \subset K_\beta$ if $\alpha > \beta$. Consequently, there exists an ordinal γ such that $K_\beta = K_\gamma$ if $\beta \geqq \gamma$. We define $K = K(T, D) = K_\gamma$ and observe that $\operatorname{clco} T(K \cap D) = K$. The mapping T is called *ultimately compact* if either $K \cap D = \emptyset$ or if $T(K \cap D)$ is relatively compact.

DEFINITION 1. Let $D \subset X$ be open with $T: \bar{D} \to K(X)$ ultimately compact and such that $x \notin T(x)$ if $x \in \partial D$. If $K(T, \bar{D}) \cap D = \emptyset$ we define $\deg(I - T, D, 0) = 0$, and if $K(T, \bar{D}) \cap D \neq \emptyset$ we let ρ be a retraction of X onto $K(T, \bar{D})$ and define

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