BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 79, Number 3, May 1973

CURVATURE AND COMPLEX ANALYSIS. III

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Communicated by S. S. Chern, October 30, 1972

This announcement is a sequel to Greene-Wu [**1**], [**2**]. Here we shall concentrate on Kähler manifolds of nonnegative curvature. Our first result improves Theorem 3 of [**2**], but the latter is needed in the proof of the former.

THEOREM 1. *Let M be a complete Kähler manifold with positive Ricci curvature and nonnegative sectional curvature. Let K be the canonical bundle of M and let L be a holomorphic line bundle on M such that $L \otimes K^* > 0$ (K^* denotes the dual of K; $L \otimes K^* > 0$ means that the line bundle $L \otimes K^*$ possesses a Hermitian metric of positive curvature). Then $H^p(M, \mathcal{O}(L)) = 0$ for $p \geqq 1$.*

The next theorem is the noncompact analogue of Kodaira's embedding theorem [**4**]. Its proof depends on Theorem 1 and is similar to Kodaira's proof in broad outline, but there are technical complications because of the noncompactness.

THEOREM 2. *Let M be a complete Kähler manifold with positive Ricci curvature and nonnegative sectional curvature. Then M possesses nonconstant meromorphic functions. Specifically, given any compact set $K \subseteq M$, there exists a positive integer N and a meromorphic mapping (see Remmert [**5**]) $\varphi : M \to P_N C$ such that $\varphi | K$ is a holomorphic embedding.*

In [**2**], we conjectured that every complete noncompact Kähler manifold with positive sectional curvature must be a Stein manifold. The next theorem includes the solution of this conjecture as a special case. Recall that a subset S of a Riemannian manifold is *convex* if, for any $p, q \in S$, at least one minimizing geodesic joining p and q lies in S.

THEOREM 3. *Let M be a complete Kähler manifold with positive Ricci curvature and nonnegative sectional curvature, and suppose that the canonical bundle of M is topologically trivial. Then every convex open subset of M is a Stein manifold.*

The fact that any open convex subset of such a manifold M is necessarily a Stein manifold should be compared with Theorem 7 of [**1**]; of course the

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AMS (MOS) subject classifications (1970). Primary 53C55, 32C10, 32E10; Secondary 32F05, 31C10.

[1] Research partially supported by National Science Foundation Grants GP-27576 and GP-34785. The second author is a Sloan Fellow and was an SRC fellow at the University of Warwick when the bulk of this work was done.