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TOPOLOGIES ON SPACES OF BAIRE MEASURES

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Introduction. Let X be a completely regular, Hausdorff space, let C be the space of real-valued continuous functions on X and let C^b be the subspace of C consisting of the uniformly bounded continuous functions on X. The Banach dual of C^b (for the uniform norm) will be denoted by M, and the subspace of M consisting of all totally-finite, signed Baire measures on X will be denoted by M_σ. (Recall that the algebra of Baire sets is the smallest σ-algebra of subsets of X for which each of the functions in C is measurable.) Finally, the space of signed measures in M_σ which have finite support will be denoted by L. By identifying each point of X with the Dirac measure at that point we may assume that X is a subset of L (and hence of M_σ). The purpose of the present note is to describe some results recently obtained by the author concerning completions of L relative to certain natural locally convex topologies on L, and some applications of these results. (For the proofs and for further details, the reader is referred to [**5**] and [**6**].) The principal results are essentially generalizations to arbitrary spaces of the following theorem due to M. Katětov and V. Pták. (See [**3**], [**4**] and [**8**].)

THEOREM. *Let X be pseudocompact. Then the completion of L for the topology of uniform convergence on the pointwise bounded, equicontinuous subsets of C is the space M_σ $(=M)$.*

In order to avoid certain technical difficulties in the discussion, we will assume throughout the paper that X has a nonmeasurable cardinal. (As is well known, it is consistent with the axioms of set theory to assume that there are no measurable cardinals.) For a discussion of the results in the presence of measurable cardinals, the reader is referred to [**5**] and [**6**].

1. **The topology e^b.** A set $B \subset C$ is *equicontinuous* if for all $x \in X$ and for every positive number ε, there is a neighborhood U of x such that $|f(x) - f(y)| \leqq \varepsilon$ for all $y \in U$ and all $f \in B$. The set B is *uniformly bounded* if there is a number K such that $|f(x)| \leqq K$ for all $f \in B$ and all $x \in X$. Let $\mathscr{E}^b$ denote the family of all uniformly bounded, equicontinuous subsets of C^b; and let e^b denote the topology on M_σ of uniform convergence on the sets in $\mathscr{E}^b$. It is easily verified that e^b is a locally convex topology on M_σ. We then have the following result:

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