BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 79, Number 3, May 1973

A REPRESENTATION FORMULA FOR LINEAR VOLTERRA INTEGRAL EQUATIONS

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Communicated by Hans Weinberger, November 1, 1972

ABSTRACT. It is shown that the solution to a finite system of linear Volterra integral equations may be expressed in terms of the fundamental solution to a so-called associated system of ordinary differential equations. The only requirement on the kernel is that it be expandable in a sufficiently convergent series of continuous matrices.

1. **Introduction.** The only goal here will be to establish a representation formula for the solution to the finite system of Volterra integral equations of the form

$$u(x) = \varphi(x) + \int_a^x K(x,t)u(t)\,dt, \qquad x_0 \leqq a \leqq x < +\infty, \tag{I}$$

in terms of the solution to a certain infinite system of ordinary differential equations. The applicability of the resultant representation formula may be varied, but we will show in future work how it leads to stability theorems for general integral equations. It may be pointed out here that the nature of the representation theorem to be proved is such that an explicit, closed-form solution may be exhibited whenever such a solution can be found for, what will be called, the associated system of ordinary differential equations.

Let $K(x,t)$ be a continuous $k \times k$ matrix ($k < +\infty$) on $x_0 \leqq t \leqq x < +\infty$, and $\varphi(x)$ be a continuous k-vector on $[x_0, +\infty)$. In a previous work [**1**], we have shown that if $U(x,s)$ satisfies the $k \times k$ matrix equation

$$U(x,s) = I + \int_s^x K(x,t)U(t,s)\,dt, \tag{U}$$

then the solution to (I) is given by

$$u(x) = U(x,a)\varphi(a) + \int_a^x U(x,s)\varphi'(s)\,ds,$$

provided we make the additional assumption that $\varphi \in C^1[x_0, +\infty)$. If $\partial U(x,s)/\partial s$ is continuous, it is easily shown that the alternate representation formula

$$u(x) = \varphi(x) - \int_a^x \frac{\partial U}{\partial s}(x,s)\varphi(s)\,ds$$

AMS (MOS) subject classifications (1970). Primary 45D05, 45M10; Secondary 45A02, 45A05, 45E99, 45F05, 45G05, 45G99, 45J05.

Key words and phrases. Volterra integral equations, representation formulas, infinite systems of ordinary differential equations.