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NONUNIFORMLY ELLIPTIC EQUATIONS: POSITIVITY OF WEAK SOLUTIONS

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1. This note is concerned with the weak boundary value problem

$$\int_\Omega \left(\sum_{i,j=1}^N a_{ij}(x) u_{x_i} v_{x_i} + b(x)uv \right) dx = \int_\Omega c(x) fv \, dx, \quad \text{all } v \in C_0^\infty(\Omega), \tag{1}$$

and the weak eigenvalue problem

$$\int_\Omega \left(\sum_{i,j=1}^N a_{ij}(x) u_{x_i} v_{x_i} + b(x)uv \right) dx = \lambda \int_\Omega c(x) uv \, dx, \quad \text{all } v \in C_0^\infty(\Omega), \tag{2}$$

where Ω is a connected open set in R^N. Our hypothesis concerning the coefficient matrix (a_{ij}) in (1) and (2) is similar to but weaker than those imposed on the elliptic operators which are studied in [**2**], [**3**], [**4**]. Specifically, we assume that $A = (a_{ij})$ is a real matrix-valued function, symmetric and positive definite almost everywhere on Ω with

$$\|A\|, \|A^{-1}\| \in L^1_{\text{loc}}(\Omega). \tag{3}$$

Concerning the coefficients b, c our assumptions are the following: b and c are real valued,

$$Mb \geqq c > 0 \quad \text{a.e. on } \Omega \tag{4}$$

for some positive constant M and

$$b, b^{-1}, c \in L^1_{\text{loc}}(\Omega). \tag{5}$$

Under these assumptions we prove: If $f \in L^2(\Omega, c(x)\,dx)$, $f(x) \geqq 0$ a.e. on Ω and $f \neq 0$ then (1) has a solution positive almost everywhere on Ω, in particular a nonnegative eigenfunction of (2) is positive almost everywhere in Ω; if (2) has a nonnegative eigenfunction corresponding to an eigenvalue $\lambda_1 > 0$ then λ_1 is simple and the spectrum of (2) is contained in the interval $[\lambda_1, \infty]$.

This research was motivated by certain problems arising in connection with the study in [**1**] of nonlinear elliptic eigenvalue problems.

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