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FORMAL GROUPS OVER DISCRETE RINGS

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In this note we develop a theory of formal schemes and groups over arbitrary commutative rings which coincides with that of [**5**] if the base ring is a field, and generalizes that of [**2**].

We assume always our base ring is discrete and treat a formal scheme (resp. group) G, with two principal tools: A topology on the affine algebra $\mathcal{O}(G)$ allows us to form its continuous linear dual $B(G)$, the coalgebra (resp. bialgebra) of distributions; a topology on C^*, the full linear dual of an arbitrary coalgebra C, allows recovery of the functor when given the distributions. In this way B establishes an equivalence between our category of formal groups and a suitable Hopf algebra category. We discuss the Verschiebung and Frobenius maps, thus illuminating some of the literature on divided powers in Hopf algebras. The results presented here will be used elsewhere for the study of curves on a formal scheme G, or equivalently, of sequences of divided powers in $B(G)$. Detailed proofs will appear elsewhere.

Throughout k is a commutative ring, algebra means commutative k-algebra, k-Alg is the category of commutative k-algebras, $\otimes$ means $\otimes_k$, etc. Func $=$ Func (k-Alg, Sets) is the category of set valued functors on k-Alg. Much terminology is standard and is collected, for example, in [**4**].

1. **The affine algebra of a functor.** Denote by $\mathcal{O}$ the underlying set functor on k-Alg. For any G in Func, the set $\mathcal{O}(G) = \mathrm{Func}\,(G, \mathcal{O})$ of natural transformations from G to $\mathcal{O}$ is an algebra via pointwise operations, functorially in G. For each A in k-Alg and each x in $G(A)$, we denote by $\chi_{x,A}$ the algebra homomorphism $\mathcal{O}(G) \to A$ given by $\chi_{x,A}(f) = f(A)(x)$. Topologize $\mathcal{O}(G)$ with the coarsest topology making each of these maps continuous. If k-Alg_c denotes the category of topological k-algebras with continuous algebra homomorphisms, then $\mathcal{O}$: Func $\to$ k-Alg_c is a functor. $\mathcal{O}(G)$ is the *affine algebra of* G.

Let $\mathcal{A}$ be a topological algebra. Write $\mathcal{A}^{\cdot}$: k-Alg $\to$ Sets for the functor $\mathcal{A}^{\cdot}(K) = k\text{-}\mathrm{Alg}_c(\mathcal{A}, K)$, regarding K with the discrete topology. Then $(\ )^{\cdot}$: k-Alg_c $\to$ Func is a functor which is adjoint on the right to $\mathcal{O}$, that is $\mathrm{Func}(G, \mathcal{A}^{\cdot}) = k\text{-}\mathrm{Alg}_c(\mathcal{A}, \mathcal{O}(G))$. If we view k-Alg as a full subcategory of

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