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LOCALLY NICE CODIMENSION ONE MANIFOLDS ARE LOCALLY FLAT

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ABSTRACT. The main result is that an $(n-1)$-manifold M in an n-manifold Q $(n \geqq 5)$ is locally flat provided $Q - M$ is locally simply connected at each point of M. Such a theorem has been obtained by Price-Seebeck in case M is locally flat at some point. This paper uses further application of their work to eliminate the additional hypothesis.

Since this note depends so heavily on the papers of Price and Seebeck, the reader is referred to [**4**], [**5**] for definitions of the terms used here. The author wishes to express his gratitude to L. S. Husch, J. G. Hollingsworth, and C. L. Seebeck for stimulating discussions.

The first lemma, an easy consequence of [**5**], is known to several people; its application to tori in Lemma 2, which is similar to Kirby's idea in [**2**], is the key to this paper.

LEMMA 1. *Suppose* $h: E^{n-1} \times \{0\} \to E^n$ $(n \geqq 5)$ *is an embedding such that* $E^n - h(E^{n-1} \times \{0\})$ *is* 1-*LC at each point of* $h(E^{n-1} \times \{0\})$ *and there exists a positive number* D *for which* $d(h, \text{incl}) < D$. *Then* $h(E^{n-1} \times \{0\})$ *is locally flat.*

PROOF. The standard contraction of E^n to the interior of the unit ball B^n shrinks $h(E^{n-1} \times \{0\})$ so that, because of the bound on the displacement of h, attaching $(E^{n-1} \times \{0\}) - \operatorname{Int} B^n$ to the image of $h(E^{n-1} \times \{0\})$ produces a manifold M (homeomorphic to E^{n-1}). Routine verification establishes that $E^n - M$ is 1-LC at each point of M, and obviously M is locally flat at points of $(E^{n-1} \times \{0\}) - B^n$. Corollary 7 of [**5**] then implies that M is locally flat, and the lemma follows.

We let T^n denote the n-dimensional torus $S^1 \times \cdots \times S^1$ (n factors).

LEMMA 2. *Suppose* h *is an embedding of* T^{n-1} $(n \geqq 5)$ *into* $T^{n-1} \times E^1$ *such that* h *is a homotopy equivalence and* $(T^{n-1} \times E^1) - h(T^{n-1})$ *is* 1-*ULC. Then* $h(T^{n-1})$ *is locally flat.*

REMARK. By appealing to [**2**, Proposition 4] we may assume that h is homotopic to the inclusion map $T^{n-1} \to T^{n-1} \times \{0\} \subset T^{n-1} \times E^1$.

PROOF. Let $p': E^{n-1} \to T^{n-1}$ and $p = p' \times 1: E^{n-1} \times E^1 = E^n \to T^{n-1} \times E^1$ denote the obvious covering maps. The existence of a homotopy

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