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ON OPERATOR-VALUED STOCHASTIC INTEGRALS

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1. **Introduction.** The purpose of this note is to announce some theorems concerning the operator-valued stochastic integrals which arise naturally from the study of the regularity properties of solutions of stochastic integral equations. Proofs and detailed discussion will be given in [**7**]. Let $B^* \subset H \subset B$ be an abstract Wiener space [**1**] and $W(t)$ a Wiener process in B. Various stochastic integrals associated with $W(t)$ were studied and an infinite-dimensional version of Ito's formula [**2**] was proved in [**5**]. This formula was used to connect the solution of a stochastic integral equation with the corresponding heat equation [**8**]. In [**6**] we proved another version of Ito's formula which was used to construct diffusion processes, in particular, a Brownian motion, in a Riemann-Wiener manifold. We present here a third version of Ito's formula and use it to prove an infinite-dimensional analogue of a formula on p. 58 of McKean's book [**9**]. An operator-valued stochastic integral has been studied by Kannan and Bharucha-Reid [**4**]. However, there appears to be no relation between their work and ours in this paper.

2. **Notation and definitions.** Let X and Y be two real Banach spaces. $L^n(X; Y)$ denotes the Banach space of all continuous n-linear maps from X^n into Y with the usual norm $\|\cdot\|_{X^n;Y}$. L^1 will be written as L. $L^{n-1}(X; X^*)$ will be identified as $L^n(X; R)$ in a well-known way. $L^2_{(2)}(H; R)$ $(\equiv L_{(2)}(H; H))$ denotes the Hilbert space of all Hilbert-Schmidt operators of H with H-S-norm $\|\cdot\|_2 = \langle\langle\, ,\rangle\rangle^{1/2}$. We have the relation $L^2(B; R) \subset L^2_{(2)}(H; R)$.

DEFINITION. Let K be a Hilbert space with inner product $(\,,)$. A continuous bilinear map S from $H \times H$ into K is said to be of *trace class type* if (i) for each $x \in K$, S_x is a trace class operator of H, where $S_x(\cdot,\cdot) = (S(\cdot,\cdot),x)$ and (ii) the functional $x \to \operatorname{trace} S_x$ is continuous. $\mathscr{S}(H; K)$ denotes the space consisting of all such continuous bilinear maps.

Let S be of trace class type. Then there is a unique element, denoted by TRACE S, of K such that (TRACE $S, x) = \operatorname{trace} S_x$ for all $x \in K$. Note that $L^2(B; L(B, B^*)) \subset \mathscr{S}(H; L_{(2)}(H; H))$.

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