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SUBMANIFOLDS, GROUP ACTIONS AND KNOTS. II

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§I contains typical examples of the application of the methods of [3], [4] to group actions. We compute equivariant knot cobordism for cyclic group fixed-point free actions. The surgery theory with coefficients that we need is outlined in §II. New algebraic K-theory functors $\Gamma_n(\Lambda' \to \Lambda)$ are introduced to solve geometric problems. For $\Lambda' = \Lambda$ these are Wall surgery groups [7].

The knot cobordism group of a manifold, or of a 2-plane bundle over a manifold is defined in §III and computed in general terms of the Γ-groups. In various cases, these Γ-groups are explicitly computed. The coefficient groups in this theory are isomorphic to the high dimensional knot-cobordism groups. The knot cobordism group of a manifold can be used to decide when sufficiently close codimension two embeddings differ, up to concordance, by a knot.

I. THEOREM 1. *Let T be a fixed-point free p.l. homeomorphism of the sphere Σ^{2k}, $k \geqq 3$, with $T^2 = 1$. Then there is at most one equivariant concordance class of invariant spheres of dimension $2k - 2$ in Σ.*

Santiago Lopez de Medrano [5] has computed which (Σ^{2k}, T) admit at least one invariant codimension two homotopy sphere; for example, this is always the case for k even.

THEOREM 2. *Let T be a fixed-point free p.l. homeomorphism of the sphere Σ^{2k+1} with $T^p = 1$, p odd. Then every fixed-point free Z_p action (S^{2k-1}, T') with S^{2k-1}/T' normally cobordant to the desuspension of Σ/T occurs, and only these occur, as the induced Z_p-actions on invariant spheres in codimension 2. If S^{2j+1} is a characteristic [5] invariant sphere of (Σ, T), there is a sequence of T-invariant spheres $S^{2j+1} \subset S^{2j+3} \subset \cdots \subset S^{2k-1} \subset S^{2k+1}$.*

Combining the above with results of Browder, Petrie and Wall [1], [8], it follows that actions induced on the invariant spheres in codimension 2 of (Σ, T), p odd, are in 1-to-1 correspondence with the elements of $Z \oplus Z \oplus \cdots \oplus Z = Z^{(p-1)/2}$.

Let T be a free action on the sphere Σ^{2k-1}, $k > 2$, with $T^n = 1$. Let $K(\Sigma, T)$ be the embeddings of (Σ, T), as an invariant subspace of a sphere

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