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AN EQUIVALENT FORMULATION OF THE INVARIANT SUBSPACE CONJECTURE

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The principal purpose of this announcement is to present an equivalent formulation of the invariant subspace conjecture for bounded linear operators acting on a Hilbert space H. Specifically, the conjecture asserts that if $B(H)$ denotes the algebra of bounded linear operators on H and $A \in B(H)$, then A has a nontrivial invariant subspace. We show that the conjecture can be reduced to the study of operators having the property that their invariant subspaces are reducing spaces. In our earlier announcement of this result we called such an operator "completely normal" (cf. [**2**]); however, since then we have been convinced (by P. R. Halmos) that "reductive" is a more appropriate term.

Throughout this note H will denote an arbitrary Hilbert space. An element $A \in B(H)$ is called *reductive* if, and only if, each invariant subspace of A reduces A. $A \in B(H)$ is called *normal-free* if there is no reducing subspace $\mathscr{M}$ for A other than (0) such that $A|_{\mathscr{M}}$ (the restriction of A to $\mathscr{M}$) is normal. If $S \subset B(H)$, then S' is the set of all $B \in B(H)$ such that $BA = AB$ and $BA^* = A^*B$ for every $A \in S$ where A^* denotes the adjoint of A. An element $C \in B(H)$ is called transitive if there are no invariant subspaces for C other than (0) and H.

Our basic result is that if $\dim H > 1$, then the invariant subspace conjecture is correct if, and only if, every reductive element of $B(H)$ is normal. Inasmuch as the proof of the result requires an elaborate use of direct integral theory for rings of operators, we have not given proofs to the theorems. The complete proofs are expected to appear in a forthcoming monograph on direct integral theory and its applications.

If $A \in B(H)$ and is reductive and $\mathscr{M}$ an invariant subspace for A then we define

$$N(\mathscr{M}) = \mathscr{M} \cap \{ A^{*n}(AA^* - A^*A)\xi \mid \xi \in \mathscr{M}, \quad n = 0, 1, \cdots \}^{\perp}.$$

In particular we set $H_0 = N(H)$ and let P_0 denote the projection of

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