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SINGULAR INTEGRALS IN THE SPACES $\Lambda(B, X)$

BY ALBERTO TORCHINSKY

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In this note we describe the theory of singular integral and multiplier transformations in the setting of the spaces $\Lambda(B, X)$. First we introduce the spaces $\Lambda(B, X)$ combining Theorems A and B below, essentially due to A. P. Calderon, with paragraphs 14 and 34 of [**2**]. Theorem C and the example that follows it illustrate the fact that $\Lambda(B, X)$ spaces are related to Lipschitz spaces of functions and distributions in R^n. (See also [**5**].) Then guided by the translation invariance of an important class of singular integrals, described before Theorem D, we define a class of operators which commute with representations of R^n into a group of (uniformly) bounded linear operators of a Banach space B into itself. The continuity of these *singular integral operators* is proved in Theorem D. Theorems E and F concerning multipliers are then proved with the assumption that the representations alluded to above are the translations. These results were submitted as a thesis at the University of Chicago. I would like to thank Professor A. P. Calderón for having had the privilege of learning with him these and many other things and to Professor Max Jodeit, Jr. for his help throughout my graduate studies.

The spaces $\Lambda(B, X)$. Let $\{t^P\}_{t>0}$ be a group of transformations of R^n where P is a real $n \times n$ matrix and

$$(*) \qquad \|t^P\| \leqq t, \qquad 0 < t \leqq 1.$$

This will ensure the existence of a unique value s for which $s^{-P}x \in S^{n-1}$, $x \neq 0$. Thus setting $\rho(x) = s$ we have that $\rho(t^P x) = t\rho(x)$ and $\rho(x + y) \leqq \rho(x) + \rho(y)$. (See [**4**].) We notice that $(Px, x) \geqq (x, x)$ is a necessary and sufficient condition for $(*)$ to hold. Moreover the adjoint matrix P^* also satisfies $(P^*x, x) \geqq (x, x)$ and therefore it determines a function $\rho^*(x)$ with similar properties. Now we construct a *one-parameter family of dilations* ν_t of a finite Borel measure ν on R^n by setting $\nu_t(E) = \nu(t^{-P}E)$ for every ν-measurable set E and $t > 0$. If $d\nu(x) = \phi(x)\,dx$ where $\phi \in L^1(R^n)$, then $d\nu_t(x) = t^{-\operatorname{tr} P}\phi(t^{-P}x)\,dx$.

Let B be a Banach space of tempered distributions on R^n such that $\mathscr{S}(R^n) \subset B$ and $B = V^*$ for some complex Banach space V. For $y \in R^n$

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