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GENERALIZED HYPERANALYTIC FUNCTION THEORY

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Lipman Bers [1] and Ilya Vekua [2] extended the concept of an analytic function by considering the weak solutions of elliptic systems of two equations, with two unknowns, and two independent variables. The solutions they investigated have come to be known as generalized (or pseudo) analytic functions. Subsequently Avron Douglis [3] introduced a class of functions which satisfy (classically) the principal part of an elliptic system of $2r$ equations, with $2r$ unknowns, and two independent variables. These functions are known as hyperanalytic functions. The present paper extends the class of functions studied by Douglis in much the same way as Bers and Vekua have extended the analytic functions. Hence, we refer to our class of functions as *generalized hyperanalytic functions.*

We shall be concerned with elliptic systems in two variables of the form

$$
\begin{aligned}
&u_{0,x} - v_{0,y} + p_{0,0}u_0 + q_{0,0}v_0 = 0,\\
&u_{0,y} + v_{0,x} + r_{0,0}u_0 + s_{0,0}v_0 = 0,\\
&u_{k,x} - v_{k,y} + au_{k-1,x} + bu_{k-1,y} + \sum_{l=0}^{k} (p_{kl}u_l + q_{kl}v_l) = 0,\\
&u_{k,y} + v_{k,x} + av_{k-1,x} + bv_{k-1,y} + \sum_{l=0}^{k} (r_{kl}u_l + s_{kl}v_l) = 0,\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad k = 1,\ldots,r-1.
\end{aligned}
\tag{1}
$$

Our work uses the hypercomplex function theory developed by A. Douglis [3] for the special case of (1) where only the principal part appears.

Douglis introduced the commutative algebra over the reals generated by the two elements i and e, subject to the multiplication rules

$$i^2 = -1, \qquad ie = ei, \qquad e^r = 0.$$

A hypercomplex number in this algebra has the form

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