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SOME NEW CONTINUITY CONCEPTS FOR METRIC PROJECTIONS

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1. **Introduction.** There has been much recent interest in studying various continuity criteria for the set-valued metric projection onto a set V. Particular interest has centered around the relationship between these criteria and either the structure of the set V itself or the geometry of the whole space. See, for example, [3], [4], [7], [8], [10], [16], [17], [18] and [21]. In essentially all of these papers, the concepts of lower semicontinuity (l.s.c.) and/or upper semicontinuity (u.s.c.) for set-valued mappings (as defined, for example, in Hahn [12]) played the key role.

In this note we introduce some simpler and more general "radial" continuity criteria. Roughly speaking, these criteria require that the restriction of the metric projection to certain prescribed line segments be l.s.c. or u.s.c. It turns out that these criteria, which are formally much weaker than l.s.c. or u.s.c., are still strong enough to generalize a number of known results, and weak enough so that many of these theorems now have valid converses (which they did not have under the stronger hypotheses of l.s.c. or u.s.c.).

Full details of proofs along with additional results and related material will appear elsewhere.

Throughout this note X will denote a (real or complex) normed linear space. For $x \in X$ and $r > 0$, let

$$S(x, r) = \{y \in X : \|x - y\| = r\}.$$

The distance from a point x to a subset V of X is defined by

$$d(x, V) = \inf\{\|x - v\| : v \in V\}.$$

The *metric projection* onto a subset V of X is the mapping P_V which associates with each $x \in X$ its set of best approximations in V, i.e.

$$P_V(x) = \{v \in V: \|x - v\| = d(x, V)\}.$$

V is called *proximinal* (resp. *Chebyshev*) provided $P_V(x)$ contains at least (resp. exactly) one point for each $x \in X$. V is called a *sun* if for each $x \in X$

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