HOW A MINIMAL SURFACE LEAVES AN OBSTACLE[1]

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ABSTRACT. We announce that the function of least area among all functions defined in a convex domain, vanishing on its boundary, and constrained to lie above a concave analytic obstacle leaves the obstacle along an analytic curve.

We announce a result about the curve of separation determined by the solution to a variational inequality. A strictly convex domain Ω with smooth boundary $\partial\Omega$ is given in the $z = x_1 + ix_2$ plane together with a smooth function $\psi(z)$ which assumes a positive maximum in Ω and is negative on $\partial\Omega$. Let K denote the closed convex set of Lipschitz functions v satisfying $v \geqq \psi$ in Ω and $v = 0$ on $\partial\Omega$. Let us denote by u the function of K which minimizes area among all functions of K; that is

$$(1) \qquad u \in K: \qquad \int_\Omega \frac{u_{x_j}}{(1 + |u_x|^2)^{1/2}} (v - u)_{x_j} \, dx \geqq 0, \qquad v \in K.$$

The existence of such u, actually satisfying $u \in H^{2,q}(\Omega) \cap C^{1,\lambda}(\bar{\Omega})$, $1 \leqq q < \infty$, $0 < \lambda < 1$, was shown in the work of H. Lewy and G. Stampacchia [7] and also in M. Giaquinta and L. Pepe [1]. For u there is a set of coincidence I consisting of the points $z \in \Omega$ where $u(z) = \psi(z)$. Let us call

$$(2) \qquad \Gamma(u) = \Gamma = \{(x_1, x_2, x_3) : x_3 = u(z) = \psi(z), z \in \partial I\}$$

the "curve" of separation.

Up to this time it has only been known that when ψ is smooth and strictly concave, Γ is a Jordan curve [2]. On the other hand, the corresponding problem for the $u \in K$ minimizing the Dirichlet integral has been thoroughly studied by H. Lewy and G. Stampacchia [6]. We wish to announce here the

THEOREM. *Let ψ be analytic and strictly concave. Let u be the solution of (1). Then $\Gamma(u)$ is an analytic Jordan curve (as a function of its arc length parameter).*

The demonstration relies on the resolution of a system of differential equations and the utilization of the system to extend analytically a con-

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