BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 6, November 1972

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CONVEX MATRIX EQUATIONS

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Communicated by Eugene Isaacson, May 15, 1972

1. **Introduction.** Let Δ_n denote the set of all $n \times n$ complex matrices A whose spectral norm $\|A\|$ is at most one. Then Δ_n forms a convex topological semigroup under matrix multiplication ([**6**], [**7**]). The subsemigroup Σ_n of Δ_n, consisting of all real nonnegative matrices in Δ_n, is the set of all $n \times n$ *doubly substochastic matrices*; that is, real nonnegative matrices whose row and column sums are at most one. The subsemigroup of Σ_n consisting of all $n \times n$ *doubly stochastic matrices* will be denoted by Ω_n.

Geometrically, Ω_n is the convex hull of the group of all $n \times n$ permutation matrices ([**1**], [**8**]), while Σ_n is the convex hull of the semigroup of all $n \times n$ subpermutation matrices [**9**]. The following theorem establishes a similar result for Δ_n.

THEOREM 1. *Δ_n is the convex hull of the set of all $n \times n$ unitary matrices.*

The proof of the theorem can be obtained by establishing that the unitary matrices form the set of extreme points of Δ_n. The result then follows by the Krein-Milman theorem. The complete proof will appear elsewhere [**10**]. Another proof of this result is given in [**15**].

Several authors have considered matrix equations involving doubly stochastic matrices. In particular, S. Sherman [**14**] and S. Schreiber [**13**] have considered the solvability of the equation $AX = B$ and D. J. Hartfiel [**5**] has considered the solvability of the equation $AXB = X$, where A, B, and X are doubly stochastic. The main purpose of this note is to consider the system of matrix equations

$$AX = B \quad \text{and} \quad BY = A, \tag{1.1}$$

AMS 1970 *subject classifications*. Primary 15A24, 15A30; Secondary 20M10, 52A40.
Key words and phrases. Convex hull, convex topological semigroup, doubly stochastic matrix, Green's relations, Moore-Penrose generalized inverse, regularity.
[1] Research supported in part by the National Science Foundation grant GP-15943.