BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 5, September 1972

ABSOLUTELY TORSION-FREE RINGS

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Communicated by Oscar Goldman, March 6, 1972

Call a ring Λ *absolutely torsion-free* (ATF) if for every finite kernel functor σ, i.e. a topologizing filter of nonzero left ideals, $\sigma(\Lambda) = 0$. In this article we note the basic properties of ATF rings, and give some related results about hereditary noetherian prime rings. The notation and terminology to be used are that of Goldman [1]. In particular, if Λ is a ring, $K(\Lambda)$ (respectively $I(\Lambda)$) will denote the set of kernel functors (respectively idempotent kernel functors) of Λ.

1. **Absolutely torsion-free rings.** We first note that ATF rings are a generalization of the familiar concept of integral domain.

PROPOSITION 1.1. *Let R be a commutative ring. Then R is an integral domain if and only if for every $\sigma \in K(R), \sigma \neq \infty \Rightarrow \sigma(R) = 0$.*

The partial ordering on $K(\Lambda)$ provides a useful description of ATF rings.

PROPOSITION 1.2. *Λ is ATF if and only if there is $\mu \in I(\Lambda)$, $\mu \neq \infty$, such that for all $\infty \neq \sigma \in K(\Lambda)$, $\sigma \leqq \mu$.*

A definition is required in order to free the concept of ATF ring from that of kernel functor. A submodule M of a module N is called a *weakly essential* submodule if for any finite subset $x_1, \ldots, x_n$ of N, there is $0 \neq r \in \Lambda$ such that $rx_i \in M$ for each i. We then have

THEOREM 1.3. *Λ is ATF if and only if every weakly essential left ideal of Λ is a rational left ideal.*

We note some elementary properties of ATF rings.

PROPOSITION 1.4. *If Λ is ATF then Λ is a prime ring, i.e. the product of nonzero ideals of Λ is nonzero. Furthermore, if R is the center of Λ, then R is an integral domain and Λ is a torsion-free R-module.*

The converse of the first part of the preceding proposition is false. For if k is a field, V an infinite-dimensional vector space over k, and

AMS 1970 *subject classifications*. Primary 16A08; Secondary 16A14.
Key words and phrases. Kernel functor, hereditary, prime ring, equivalence of categories, maximal ring of quotients, simple ring, von Neumann regular, selfinjective, Goldie ring, epimorphism of rings, flat module.