BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 5, September 1972

ON A CLASS OF COMPLEX SPACES INTERMEDIATE TO STEIN AND COMPACT

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Communicated by Mary Ellen Rudin, March 20, 1972

In the global theory of complex spaces two extreme cases were traditionally studied: Compact and Stein. A pseudoconcave space represents a variation from this tradition. However, from the point of view of global function theory, pseudoconcave spaces behave much like compact spaces. For example, on a pseudoconcave space every holomorphic function is constant and the meromorphic function field is an algebraic function field having transcendence degree over C bounded by the dimension of the space [1].

The q-pseudoconvex (concave) case, which was investigated by Andreotti and Grauert [2], represents another variation. However, the main result in the q-pseudoconvex case is that $\dim_C H^r(X, \mathscr{F}) < \infty$ for $\mathscr{F}$ coherent and $r \geqq q$ [2]. Unless X is Stein, nothing can be said about the lower cohomology groups or, in particular, the algebra of holomorphic functions. The main result in the q-pseudoconcave case is another finiteness theorem [2], but, as we have already remarked, pseudoconcave spaces behave much like compact spaces from the function theoretic standpoint.

We are therefore led to study complex spaces which at least from the global function theoretic point of view, are more clearly between the extremes of Stein and compact spaces.

DEFINITION. Let X be a complex space. We say that X is *k-pseudoflat* if there is a relatively compact open subset $Y \subset X$ such that every $p \in \partial Y$ is contained in a set $S_p \subset \bar{Y}$, where S_p is an analytic subvariety of an open set in X such that $\operatorname{codim}_p S_p \leqq k$.

Of course the motivation for such a definition is a complex manifold containing a relatively compact open subset with smooth boundary which is Levi flat in a certain number of tangent directions.

In [4] we prove the following theorem.

THEOREM 1. *Let X be a reduced, irreducible k-pseudoflat complex space. Then there are at most k analytically independent holomorphic functions on X.*

AMS 1969 *subject classifications*. Primary 3222, 3247.
Key words and phrases. Pseudoflat, Levi flat, holomorphic functions, analytic independence, complex Lie groups.
[1] Partially supported by NSF grant GP-20139.