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ESTIMATES FOR THE SZEGÖ AND POISSON KERNELS OF SUFFICIENTLY ROUNDED TUBE DOMAINS

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In this paper we obtain estimates for the decrease at infinity of the Szegö and Poisson kernels, $S_\Gamma(X, Y) = S_{\Gamma;X}(Y)$ and $P_\Gamma(X, Y) = P_{\Gamma;X}(Y) = |S_\Gamma(X, Y)|^2 \|S_{\Gamma;X}\|_2^{-2}$, associated with proper cones $\Gamma \subset \boldsymbol{R}^n$ which are sufficiently smooth and satisfy certain curvature conditions. These estimates verify, for these cases, the conjecture of Stein (see [2], [4]) that the Poisson integral of an L^1 function converges restrictedly almost everywhere to that function on the distinguished boundary of a tube domain (Corollary IA). These and other results about the Poisson kernel will be elaborated on in [1].

Let Γ be a proper cone of $\boldsymbol{R}^n$ (that is, a nonempty, open, convex cone whose closure contains no whole line), Γ^* its dual cone

$$(1) \qquad \Gamma^* = \{Y \in \boldsymbol{R}^n : (X, Y) > 0\ \forall X \in \bar{\Gamma} - \{0\}\},$$

which is also proper, and $\Omega = \Omega_\Gamma$ its tube domain

$$(2) \qquad \Omega = \Gamma \times i\boldsymbol{R}^n = \{Z \in \boldsymbol{C}^n : \mathrm{Re}(Z) \in \Gamma\}.$$

For $X \in \Gamma$ define the nonempty compact section $C_{\Gamma^*;X} = C_X^*$ of $\bar{\Gamma}^*$ as follows:

$$(3) \qquad C_X^* = \{Y \in \bar{\Gamma}^* : (X, Y) = 1\} \subset \{Y : (X, Y) = 1\} \approx \boldsymbol{R}^{n-1};$$

and similarly for $C_{\Gamma;Y} = C_Y$, $Y \in \Gamma^*$.

We will say Γ is C^N, $N \geqq 0$, if ∂C_Y is C^N. Γ will be said to satisfy the "flat curvature condition" if for some proper circular cone Δ of $\boldsymbol{R}^n$ and every $P \in \partial\Gamma$ there is a rotation ρ_P of $\boldsymbol{R}^n$ such that $P \in \partial(\rho_P\Delta)$ and $\rho_P\Delta \subset \Gamma$. The dual condition, the "sharp curvature condition," is stated similarly but reverses the last inclusion. We exclude, in our theorems, the trivial cases $n = 1, 2$.

THEOREM I. *Suppose Γ is a proper cone of $\boldsymbol{R}^n$, where*
(a) *$n = 3$ and Γ satisfies the flat curvature condition, or*
(b) *$n \geqq 4$, Γ is $C^{[n/2]}$, and Γ satisfies the sharp curvature condition.*

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