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PUNCTUAL HILBERT SCHEMES[1]

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This note is about the structure of families of open ideals in the ring of power series in two variables. The Hilbert scheme parametrizing them is stratified into locally closed subschemes Z_T, whose dimension we calculate. We then discuss some global consequences for families of 0-dimensional schemes on a surface (§1, Corollaries 2, 3). Except in low characteristics, Z_T is locally an affine space (Theorem 2) and is a locally trivial bundle over the complete variety G_T parametrizing graded ideals of type T (Theorem 3).

1. **A stratification of the Hilbert scheme.** Let R be the ring of power series $k[[x_1, \ldots, x_r]]$ in r variables over an algebraically closed field k, with maximal ideal m; and let R_j denote the space of forms of degree j in R, so that $R = \prod R_j, j = 0, \ldots, \infty$. If I is an ideal in R, we let I_j denote the space of forms in R_j which are initial forms of elements of I. By the *type of I* we mean the sequence

$$T(I) = (t_0, t_1, \ldots, t_j, \ldots), \quad \text{where } t_j = \dim_k(R_j/I_j). \tag{1}$$

We will sometimes refer to a type T, meaning a specific infinite sequence $(t_0, t_1, \ldots)$. By the *length* $|T|$ of T we mean $\sum t_j$, if it is finite. The *initial degree of I* is the smallest j for which $I_j \neq 0$. It depends only on the type of I. It is easy to show that if I has finite colength n, then $n = |T(I)|$, and $t_j = 0$ if $j \geqq n$.

Let $\mathrm{Hilb}^n R$ be the Hilbert scheme parametrizing the family of ideals of colength n in R, and Z_T the subscheme parametrizing ideals of a given type T where $|T| = n$. Then we get a stratification (see [7])

$$\mathrm{Hilb}^n R = \bigcup_{|T| = n} Z_T.$$

For the rest of the paper we consider the case $r = 2$, and let $A = k[[x, y]]$. If $I \subset A$ has colength n and initial degree d, then

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