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PURE, NORMAL MAXIMAL SUBFIELDS FOR DIVISION ALGEBRAS IN THE SCHUR SUBGROUP

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In this paper we shall give very simple relations which define those division algebras contained in the Schur subgroup of the Brauer group of a field F of characteristic zero.

Any irreducible representation of a finite group over a field of characteristic zero corresponds to a simple component $\mathfrak{A}$ in the group algebra over that field. The character afforded by the representation will have as constituent an irreducible complex character χ, and the center F of $\mathfrak{A}$ will contain the values of χ on the group. Decompose $\mathfrak{A}$ as $\mathfrak{D}\otimes\mathfrak{M}$, where $\mathfrak{D}$ is a division algebra and $\mathfrak{M}$ is a full matrix algebra, both with center F. All of the finite-dimensional division algebras with center F form an Abelian group, called the Brauer group of F. Those division algebras obtained by the method just described form a subgroup, the Schur subgroup of F. The dimension of $\mathfrak{D}$ over F is the square of an integer m which is called the Schur index of χ. It has recently been proved by M. Benard and M. Schacher [2] that F contains the mth roots of unity. The purpose of this note is to draw attention to the following interesting consequence of this result.

THEOREM. *Let $\mathfrak{D}$ be the division algebra appearing in the factorization of a simple component of the group algebra of a finite group over a field of characteristic zero. Then $\mathfrak{D}$ is generated over its center F by elements A and B satisfying the relations*

(1) $A^{-1}B^{-1}AB = \varepsilon$,

(2) $A^m \in F$,

(3) $B^m \in F$,

where m is the index of $\mathfrak{D}$ and ε is a primitive mth root of unity.

The fields $K = F(A)$ and $L = F(B)$ are pure maximal subfields of $\mathfrak{D}$ which are normal extensions of F with cyclic Galois group.

We begin our proof with the observation that there is a division algebra $\mathfrak{D}_0$ in the rational group algebra such that $\mathfrak{D} = \mathfrak{D}_0 \otimes F$. This is a tensor product over the center $Q(\chi)$ of $\mathfrak{D}_0$ where χ is some irreducible complex character afforded by the group. The fundamental structure theorem [1,

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