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INVOLUTIONS ON KLEINIAN GROUPS

BY I. KRA[1] AND B. MASKIT

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The purpose of this note is to give an elementary proof of the following result.

THEOREM A. *Let G be a finitely generated nonelementary Kleinian group and let J be an anticonformal homeomorphism of $\Omega = \Omega(G)$, the set of discontinuity of G, where J commutes with every element of G. Then J is the restriction of an anticonformal, involutory fractional linear transformation (that is, $J(z) = (a\bar{z} + b)/(c\bar{z} + d)$, $J^2 = 1$) and G is either Fuchsian or a Z_2-extension of a Fuchsian group. Further, the mapping J with the above properties is unique.*

We prove Theorem A by reducing it to

THEOREM B. *Let Γ be a finitely generated Fuchsian group operating on U_1 and U_2, the upper and lower half-planes, respectively. Let f_1 and f_2 be schlicht functions on U_1 and U_2, where $f_1 \circ \gamma \circ f_1^{-1}$ and $f_2 \circ \gamma \circ f_2^{-1}$ both define the same isomorphism of Γ onto a Kleinian group G, and $f_1 = f_2$ on that part of the real axis $\boldsymbol{R}$ lying in $\Omega(\Gamma)$. Then f_1 and f_2 are restrictions of the same fractional linear transformation.*

As a corollary to our proof of Theorem B, we obtain the somewhat more general

THEOREM C. *Let Γ be a finitely generated Fuchsian group of the first kind acting on U_1 and U_2. Let f_1 defined on U_1, and f_2 defined on U_2 be holomorphic cover mappings where $f_1 \circ \gamma \circ f_1^{-1}$ and $f_2 \circ \gamma \circ f_2^{-1}$ both define the same homomorphism of Γ onto a Kleinian group G. Then G is either Fuchsian or a Z_2-extension of a Fuchsian group (perhaps of the second kind).*

REMARK. Theorem C gives information about certain deformations of Γ, in the sense of Kra [**6**], where the same deformation is supported in both U_1 and U_2. Nothing is known about the more general case where f_1 and f_2 are merely locally schlicht.

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