

PL SHEAVES AND THEIR CHARACTERISTIC CLASSES

BY HOWARD OSBORN

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ABSTRACT. Any PL manifold possesses a natural structure sheaf and a derivation into a sheaf \mathcal{E} of differential forms, from which one obtains the smoothable function algebra and de Rham complex of [1] via global sections. The sheaves \mathcal{E} are part of a fibered category of sheaves \mathcal{F} of modules over PL structure sheaves, into which the classical category of (sheaves of local sections of) vector bundles embeds as a full subcategory. There is a Chern-Weil construction of real characteristic classes which assigns Chern classes to complex sheaves \mathcal{F} and Euler classes to real oriented sheaves \mathcal{F} in such a way that all the usual axioms are satisfied. These classes are precisely the usual real Chern and Euler classes on the subcategory of vector bundles.

In this note we present definitions and statements of some of the main results concerning PL sheaves and their real characteristic classes. The details will appear in [2].

1. **Antesheaves.** The data one needs to describe certain sheaves on PL manifolds arise most naturally in the form of *antesheaves*, defined in this section. The definition presented here is a slight variant of the definition used in [2]; the resulting sheaves are the same, however.

For any positive integer $p \leq n$ an open p -simplex of R^n at 0 is the set of points $x_1 e_1 + \cdots + x_p e_p$ for fixed linearly independent vectors $e_1, \dots, e_p \in R^n$ and all positive x_1, \dots, x_p ; the open 0-simplex of R^n at 0 is $\{0\}$. A simplicial decomposition of R^n at 0 is any decomposition of R^n into a disjoint union of finitely many open p -simplexes, such that each of the $2^p - 1$ open simplexes occurring as faces of any open p -simplex in the decomposition also belongs to the decomposition. By translation one defines simplicial decompositions of R^n at any point $P \in R^n$. Finally, if U is any open set of R^n a *simplicial decomposition α of U at $P \in U$* consists of the intersections with U of the open p -simplexes of a simplicial decomposition of R^n at P .

For any simplicial decomposition α of U at $P \in U$ and any $Q \in U$ the *open star set of Q in α* is the union of those open simplexes of α whose closures contain Q .

Now suppose that an n -dimensional PL manifold M is described by an atlas $\{(U_i, \Psi_i)\}$, where $\{U_i\}$ is an open cover of M , and where each Ψ_i is a homeomorphism of the corresponding U_i onto an open set

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