BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 5, September 1972

ON THE EXISTENCE OF A "WAVE OPERATOR" FOR THE BOLTZMANN EQUATION[1]

BY F. ALBERTO GRÜNBAUM

Communicated by Harry Kesten, February 1, 1972

ABSTRACT. The Boltzmann equation is considered on the appropriate Hilbert space. The nonlinear problem is looked at as a perturbation of its linearized version. Thus, one deals with a pair of contractive semigroups, and a "wave operator" for this pair is studied. We find a subspace of finite codimension where the corresponding limit exists.

The Boltzmann equation for a monoatomic gas is

$$\partial f/\partial t + v_1 \cdot \operatorname{grad} f = Bf$$

$$(1) \qquad = \iint [f(v_2^*)f(v_1^*) - f(v_2)f(v_1)] \cdot |v_1 - v_2| I(|v_1 - v_2|, \theta) \sin\theta \, d\theta \, d\phi \, dv_2.$$

Here $f(t, r, v)$ is the velocity distribution function at time t at the point r, and the star on v_1 and v_2 denotes the effect of a binary collision. $I(|v_1 - v_2|, \theta)$ is the differential scattering cross section corresponding to the turning of the relative velocity $v_1 - v_2$ in an interaction.

We are concerned with the spatially homogeneous case and moreover we assume that we are dealing with a cut-off interaction, so that

$$(2) \qquad \int I(v, \theta) \sin\theta \, d\theta \, d\phi < \infty.$$

Under these restrictions the initial value problem for the Boltzmann equation has been much studied.

There is one molelular interaction, proposed by Maxwell, which simplifies the mathematics in (1) a bit. One proposes a central potential inversely proportional to r^4 and one finds that $vI(v, \theta)$ is a function of θ alone, with a pole at $\theta = 0$. This pole is removed by the cut-off assumption (2). Thus the equation can be written as

---

AMS 1970 *subject classifications*. Primary 45M05, 47H15, 82A40; Secondary 33A65, 60K35.

Key words and phrases. Boltzmann equation, nonlinear equation, linearized version, contractive semigroups, wave operator, heat flow vector.

[1] The research reported in this paper was initiated at the Rockefeller University, and completed at New York University with the support of the U.S. Atomic Energy Commission, Contract No. AT(30-1)-1480.