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ON ALGEBRAIC VARIETIES WHOSE UNIVERSAL COVERING MANIFOLDS ARE COMPLEX AFFINE 3-SPACES

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1. **Introduction.** Let V be a nonsingular projective algebraic variety defined over the field of complex numbers. By $\tilde{V}$ we denote the universal covering manifold of V. It is clear that if $\tilde{V}$ is an abelian variety, then $\tilde{V}$ turns out to be a complex affine space. The author is concerned with a converse of this fact. Thus, he proposes the following:

CONJECTURE U_n. Suppose that $\tilde{V}$ is a complex affine n-space. Then there exists an abelian variety which is a finite unramified covering manifold of V.

This has been solved only for $n = 1, 2$. We note that the proof for $n = 2$ requires a detailed study of the classification of algebraic surfaces. In his thesis [3], the author introduced the notion of Kodaira dimension $\kappa(V)$ of algebraic varieties V and by using it he intends to extend the classification theory into higher dimensional case (see [5]). In this note, he shall give a sketchy proof of the following partial solution of U_3.

THEOREM. *Suppose that V satisfies the hypothesis for U_3. Then $\kappa(V) \neq 1$ and* 3.

The detailed proof and related results will appear elsewhere.

2. **Divisor-dimension and Kodaira dimension.** We recall definitions and some results concerning divisor-dimension and Kodaira dimension (see [3]). Let V be a complete algebraic variety and D a Cartier divisor on V. Denoting by $\mathcal{O}(D)$ the invertible sheaf associated with D, we define $l(D)$ to be $\dim \Gamma(V^*, \mu^*\mathcal{O}(D))$ where $\mu: V^* \to V$ is a normalization of V. We study $l(mD)$ as a function of m for sufficiently large integer m. If there exists a positive integer m_0 such that $l(m_0 D) > 0$, we can find real positive constants α, β and a nonnegative integer κ which satisfy

$$\alpha m^{\kappa} \leqq l(m \cdot m_0 D) \leqq \beta m^{\kappa}$$

for sufficiently large values of m. Since the κ is independent of the choice

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