BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 5, September 1972

PHYSICAL VARIATIONAL PRINCIPLES WHICH SATISFY THE PALAIS-SMALE CONDITION

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Communicated by Philip Hartman, February 9, 1972

1. Using variational techniques we have found conditions which insure the existence of trajectories to conservative dynamical systems which wind around singularities of the potential and are of the following four types: (1a) periodic trajectories cutting arbitrary small neighborhoods of the singularities; (1b) periodic trajectories having arbitrary given period; (2a) trajectories joining two fixed points and cutting arbitrarily small neighborhoods of the singularities; (2b) trajectories joining two fixed points with arbitrary given time of transit.

The trajectories are obtained as paths at which certain functionals attain extreme values. These functionals satisfy the Palais-Smale condition, and the trajectories are computable by the Ritz method. At present we must impose a certain condition on the potential which excludes the gravitational case, and the P-S condition is definitely not satisfied by these functionals in this case. Moreover, our theorems provide for the existence of trajectories of a type which cannot occur in the gravitational case.

On the other hand, our theorems apply to planar n-body systems consisting of particles which attract each other with forces that are (roughly) inverse cube or stronger in a neighborhood of each particle. Hence, e.g., we obtain periodic solutions to such systems which are very complicated but have arbitrarily long or short periods.

2. Consider the dynamical equation

$$\ddot{x} + \nabla V(x) = 0, \tag{1}$$

where $x = (x^1, \ldots, x^N)$ denotes a general point of R^N and $V = V(x)$ is a real valued function on R^N with gradient ∇V. It is assumed that V is of class C^2 everywhere on R^N except at a nonempty closed set of points S at which V has infinitely deep wells; i.e., we suppose that $V(x) \to -\infty$ as $x \to S$. We shall at times assume one or more of the following conditions. (The first condition is always assumed, and this is the condition which excludes the gravitational case. See §4.)

AMS 1970 *subject classifications*. Primary 70H25, 58F05; Secondary 58E99.

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