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FROBENIUS AND THE HODGE FILTRATION

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The problem of p-adically estimating the number of solutions of an algebraic variety over a finite field of characteristic p was initiated by the classical result of Chevalley-Warning [**11**].

THEOREM. *Let $G(x_1, \ldots, x_n)$ be a polynomial with integral coefficients, of degree less than n. The number of solutions of*

$$G(x_1, \ldots, x_n) \equiv 0 \mod p$$

is divisible by p.

My aim in this paper, is to explain a conjecture of N. Katz [**3**], which provides quite sharp information concerning this general problem, and to indicate some theorems I have obtained which affirm his conjecture, under a mild hypothesis. Proofs of these theorems will be contained in [**8**].

1. **The Zeta-function of a variety over a finite field.** Let X_0 be a scheme of finite type over $k = F_q$. We wish to study N_s, the number of rational points of X_0 over F_{q^s}, for all $s > 1$. The Zeta-function of X_0/k expresses this information for us:

$$Z(T; X_0/k) = \exp\left(\sum_{s=1}^{\infty} \frac{N_s T^s}{s}\right).$$

Given any "Weil cohomology" in the terminology of Kleiman's survey article [**6**], the Zeta-function may be expressed as an alternating product of characteristic polynomials,

$$Z(T; X_0/k) = \prod_i \det(1 - f_i T)^{(-1)^{i+1}}$$

where f_i denotes the endomorphism of $H^i(X_0)$ induced by f, the qth power endomorphism of the structure sheaf of X_0. Moreover, if X_0 is proper and smooth, the Zeta-function satisfies a functional equation with respect to a change of variables

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