BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 4, July 1972

A CENTRAL LIMIT THEOREM FOR A CLASS OF d-DIMENSIONAL RANDOM MOTIONS WITH CONSTANT SPEED[1]

BY LUIS G. GOROSTIZA

Communicated by Harry Kesten, February 7, 1972

Consider the motion of a point in a d-dimensional Euclidean space R^d ($d \geqq 1$), which starts out from the origin at the time zero in a random direction with constant speed, and at the end of a random time starts afresh from its position with a direction which is a random orthogonal transformation of the previous direction, and so on for all time. The position of the point in time defines a d-dimensional random process with continuous trajectories.

Consider a sequence of such processes; specifically, for $n = 1, 2, \ldots,$

$$X_n(t) = c_n \sum_{i=1}^{N_n(t)} \tau_{n,i} A_{n,i-1} \cdots A_{n,0} \xi_n + c_n\left(t - \sum_{i=1}^{N_n(t)} \tau_{n,i}\right) A_{n,N_n(t)} \cdots A_{n,0}\xi_n, \qquad t \geqq 0,$$

where c_n is a positive constant (the speed); $\tau_{n,i}$, $i \geqq 1$, are nonnegative, nonidentically zero random variables (the times between direction changes); $N_n(t) = \max\{k : \sum_{i=1}^k \tau_{n,i} \leqq t\}$, $t \geqq 0$; $A_{n,i}$, $i \geqq 1$, are random orthogonal matrices (the changes of direction); $A_{n,0} = I$; and ξ_n is a random unit vector (the initial direction). For each n we assume that $\tau_{n,i}$, $i \geqq 1$, $A_{n,i}$, $i \geqq 1$, and ξ_n are independent, and that $\tau_{n,i}$, $i \geqq 1$, are identically distributed, as are $A_{n,i}$, $i \geqq 1$. I denotes the identity matrix.

To obtain an invariance principle for such sequences of processes having a Gaussian limit we make the following further assumptions. In order that the times between direction changes tend to zero as $n \to \infty$, we suppose there are random variables τ_i and positive constants b_n such that $\tau_{n,i} = \tau_i/b_n$ and $\lim_{n\to\infty} b_n = \infty$. To prevent the point from remaining on a proper subspace of R^d, $d \geqq 2$, or from traveling in only one direction in R^1, we require the random orthogonal matrices $A_{n,i}$ to be irreducible, the definition of an irreducible random matrix of order d being that it has no almost surely invariant nontrivial subspace if $d \geqq 2$, and that it is not

AMS 1970 *subject classifications*. Primary 60F05; Secondary 60K99.
Key words and phrases. d-dimensional random process, Gaussian process, weak convergence.

[1] This result is contained in the author's 1972 Ph.D. dissertation at the University of California at Los Angeles, written under Professor Charles J. Stone. This research was supported by the Instituto Politécnico Nacional and the Comisión Nacional de Energía Nuclear, Mexico.