

A REPRESENTATION OF A POSITIVE LINEAR MAPPING

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Let X and Y be compact Hausdorff spaces. Let $C(X)$ and $C(Y)$ be the algebras of real valued continuous functions on X and Y respectively. $C(X)$ and $C(Y)$ are endowed with their natural partial ordering and their sup norm. Let $\Phi: C(X) \rightarrow C(Y)$ be a positive, bounded linear mapping.

X is said to have the Souslin property if every disjoint family of non-empty open subsets of X is countable.

A lattice L is said to satisfy the countable chain condition upward if the following is true: For any upper bounded subset A of L , there exists a countable subset B of A such that A and B have the same family of upper bounds. The countable chain condition downward on a lattice can be defined in a similar fashion.

A lattice L is said to satisfy the countable chain condition if L satisfies both the countable chain condition upward and the countable chain condition downward.

The purpose of this note is to announce the results on representation for Φ , based on the techniques developed in [1], [2].

To get the main theorem, we need the following series of propositions which are interesting in themselves.

PROPOSITION. *For a given compact Hausdorff space X , there exists a complete Boolean space X^* and a mapping $\sigma: C(X) \rightarrow C(X^*)$ such that σ is an isometric, order preserving and algebra monomorphism.*

REMARK. The construction of σ here is different from the one in [3]. A part of the proof comes from an application of the Gelfand-Naimark theorem [4].

We study a necessary and sufficient condition on X under which $C(X^*)$ satisfies the countable chain condition so that we later use this result to represent Φ as the Maharam integral [2].

To this end, we introduce the concept of the countable chain condition on a Boolean algebra [6] and the pseudocountable chain condition on $C(X)$.

$C(X)$ is said to satisfy the pseudocountable chain condition if every disjoint set of nonzero elements of $C(X)$ is countable. (Two functions f and g of $C(X)$ are disjoint if $\inf(f, g) = 0$.)

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