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BASIC ELEMENTS: THEOREMS FROM ALGEBRAIC K-THEORY

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1. **Introduction.** Several of the fundamental theorems about algebraic K_0 and K_1 are concerned with finding unimodular elements, that is, elements of a projective module which generate a free summand. In this announcement we use the notion of a *basic element* (in the terminology of Swan [5]) to extend these theorems to the context of finitely generated modules which may not be projective. Our techniques allow a simplification and strengthening of the existing results even in the projective case. Our Theorem A includes extensions of Serre's theorem on free summands of "large" projective modules, Bass' cancellation and stable range theorems, and the theorem of Forster and Swan on the number of generators of a module. Theorem B contains a further extension of the Forster-Swan theorem and the stable range theorem.

The proofs of our theorems, which will appear elsewhere, involve a mixture of the methods of Swan [5] and Bass [1, Theorem 8.2]. By making heavy use of the generic points provided by the j-spectrum, Swan's method enables one to simplify Bass' proofs and obtain stronger results.

2. **Preliminaries.** Throughout this paper, rings have units and modules are finitely generated. R will always denote a commutative ring with noetherian maximal spectrum, and A will denote an R algebra which is finitely generated as an R-module.

The theorems of Bass and Serre that we have mentioned were proved using only the space of maximal ideals of R. To extend these results, it is necessary to work with all the j-primes of R; that is, those primes of R which are intersections of maximal ideals. (See [5] for a detailed exposition of the relation with the maximal spectrum.) Following Swan [5] we will say that the j-dimension of a j-prime is the length of the longest chain of j-primes containing it. Other j-concepts are defined similarly. The fundamental notions of this paper are the following:

DEFINITION. Let M be an A-module, $m \in M$, and x a prime ideal of R. *m is basic in M at x* if m can appear as part of a minimal system of generators for the A_x-module M_x (in particular $M_x \neq 0$).

m is j-basic in M if m is basic in M at every j-prime of R.

If $M' \subset M$ is a submodule, then *M' is n-fold basic in M* at x if the minimal

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